Defence Research and Development Canada
Recherche et développement pour la défense Canada

DEFENCE R&D DÉFENSE

Impact of the tactical picture quality on the fire control radar search-lock-on time

F. Rhéaume
A. R. Benaskeur
DRDC Valcartier

Defence R&D Canada – Valcartier
Technical Report
DRDC Valcartier TR 2003-284
September 2006

Canada

Impact of the tactical picture quality on the fire control radar search-lock-on time

F. Rhéaume
A. R. Benaskeur
Defence R&D Canada - Valcartier

Defence R&D Canada - Valcartier

Technical Report

DRDC Valcartier TR 2003-284

September 2006

Principal Author

F. Rhéaume

Approved by

Éloi Bossé
Head/A Section

Approved for release by

G. Bérubé
Chief Scientist

Abstract

Data fusion is suitable for a broad range of decision support applications. To cope with a larger class of problems and contexts, data fusion gains to be adaptive. Adaptation in data fusion corresponds to Level 4 of the JDL model, also referred to as process refinement. The Decision Support Systems Section (DSS) at Defence Research & Development Canada (DRDC) – Valcartier has initiated research activities aiming at developing and demonstrating advanced concepts of adaptive data fusion that could apply to the current Halifax and Iroquois Class Command & Control Systems (CCS), as well as their possible future upgrades, in order to improve their performance against the predicted future threats. This document gives a brief description of the adaptive data fusion concepts. It also presents a new Measure Of Effectiveness (MOE) that serves as an adaptation trigger in the target-tracking problem in maritime Above Water Warfare (AWW) applications. The proposed MOE uses the search to lock-on time of the Fire Control Radar (FCR) and aims at establishing and quantifying the effect of the quality of the Maritime Tactical Picture (MTP) on the diminution of battle space size and reaction time. Besides adaptation of the sensing and processing operation, this MOE allows addressing the trade-off between the time dedicated to the tracking with surveillance radars versus the time spent in search and lock-on with FCR.

Résumé

La fusion de données se prête à une vaste gamme d'applications pour l'aide à la décision. Afin de pouvoir répondre à une classe étendue de problèmes, la fusion de données gagnerait à être adaptative. Ceci définit le niveau 4 du modèle JDL de fusion de données. La Section des systèmes d'aide à la décision (SAD), à Recherche et développement pour la défense Canada (RDDC) - Valcartier, a initié des activités de recherche ayant comme principal objectif de démontrer des concepts avancés de fusion adaptative des données. Ces concepts pourraient s'appliquer aux systèmes actuels de commandement et contrôle des navires canadiens de classe Halifax et Iroquois, ainsi qu'à leurs versions futures. Ce document fournit une brève description des concepts de fusion adaptative de données. Également, une nouvelle mesure d'efficacité est définie et présentée. Cette mesure servirait de détecteur de changement dans la situation pouvant activer l'adaptation. Cette mesure d'efficacité emploie le temps de recherche du radar de conduite de tir et vise à établir et quantifier l'effet de la qualité de l'image tactique maritime sur la diminution du temps de réaction. Cette mesure d'efficacité permet également d'aborder le problème de la recherche de compromis entre le temps alloué au pistage par les radars de surveillance et au radar de conduite de tir pour la recherche.

This page intentionally left blank.

Executive summary

Impact of the tactical picture quality on the fire control radar search-lock-on time

F. Rhéaume, A. R. Benaskeur ; DRDC Valcartier TR 2003-284; Defence R&D Canada - Valcartier; September 2006.

Adaptive data fusion is aimed at developing systems that can cope with changes in their context by performing structural modifications. This constitutes a step further compared to the purely passive and open loop data fusion. It allows obtaining increased quality with equal or less effort. Adaptive systems can be seen meeting adaptation requirements that are: goal management, performance evaluation, effectiveness evaluation, and action selection. These requirements should enable the system to automatically adapt to unforeseen and changing contexts, thus resulting in better responses.

This report describes research activities conducted at Defence Research & Development Canada (DRDC) – Valcartier that aims at bringing contributions in adaptive data fusion area on both theoretical and experimental sides. The objective of the reported work is to define a new Measure Of Effectiveness (MOE) that serves for the adaptation of the target tracking operation in maritime Above Water Warfare (AWW) context. Measures of Effectiveness are required to evaluate the system behaviour with respect to high level and operational objectives. The results of such an evaluation set the low level objectives. The proposed MOE uses the search and lock on time of the Fire Control Radar (FCR) and aims at establishing and quantifying the effect of the quality of the Maritime Tactical Picture (MTP) on the diminution of battle space size and reaction time. This work attempts to demonstrate the benefits of adaptive tracking that makes use of reaction time as a context compared to the classical open loop tracking mode.

The Fire Control Radar is involved in the target engagement phase of the maritime AWW operations. Once a decision is made to engage a given target, the FCR is cued with positional information from the surveillance-sensor-trackers in order to re-acquire, track and engage it. The FCR has a search time restricted by the reaction time that depends on several factors such as the ownship weapons properties, Command & Control System (CCS) performance, the operator skill/training, and by the attacking target characteristics. Besides these factors, the time available to the decision is affected by the duration of search and lock-on operation. This time the FCR will take to re-acquire the target is directly dependent on the size of uncertainty volume defined in the tactical picture. A poor quality MTP causes the FCR to search in a large size volume. Such an MTP will have grave consequences on the battle space size, since the time for the search and lock-on depends, in a non-linear manner, on the volume to be scanned and the probability of detection of the FCR. This time is subtracted from the total reaction time necessary to the ship's survival. This search to lock-on time constitutes the new Measure Of Effectiveness proposed in this document. A relationship between the track quality and the diminution of battle space size and reaction time is defined.

Sommaire

Impact of the tactical picture quality on the fire control radar search-lock-on time

F. Rhéaume, A. R. Benaskeur ; DRDC Valcartier TR 2003-284; R&d pour la défense Canada - Valcartier; septembre 2006.

La fusion adaptative de données vise à développer des systèmes capables de répondre adéquatement aux changements de leur contexte. Ce concept constitue une évaluation par rapport à la fusion passive et en boucle ouverte. Les systèmes adaptatifs peuvent être perçus comme répondant aux exigences d'adaptation que sont la gestion des buts, l'évaluation de la performance, l'évaluation de l'efficacité et la sélection des suites d'actions.

Ce rapport décrit les activités de recherche, menées à Recherche et développement pour la défense Canada (RDDC) – Valcartier, qui visent à apporter une contribution au domaine de la fusion adaptative, tant sur le plan théorique qu'expérimental. L'objectif du travail présenté dans ce document est de proposer une nouvelle mesure d'efficacité qui peut agir comme déclencheur d'adaptation dans les applications de pistage de cible. Pour les systèmes adaptatifs, les mesures d'efficacité sont souvent requises pour évaluer les performances du système par rapport aux objectifs de haut niveau. Le résultat de cette évaluation établit les objectifs de niveau inférieur. La mesure d'efficacité proposée utilise le temps de recherche du radar de conduite de tir et vise à établir et à quantifier la relation de dépendance entre la qualité de l'image tactique et la diminution de temps de réaction. Le travail présenté tente de démontrer l'avantage du pistage adaptatif qui utilise le temps de réaction comme contexte, comparé au pistage classique en boucle ouverte sans rétroaction.

Dans le cadre de guerre anti-aérienne et de surface, le radar de conduite de tir fait partie de la phase d'engagement des cibles. Une fois la décision prise pour engager une cible, ce radar reçoit du système de pistage les informations cinématiques de la cible. Il entame alors, sur la base de cette information, une opération de recherche afin de réacquérir et pister la cible à son tour pour fins d'engagement. Le temps de recherche du radar de tir est limité par le temps de réaction qui, à son tour, dépend de plusieurs facteurs. Il faut noter que cette dépendance est réciproque. Le temps de réaction disponible au décideur est également affecté par la durée de la recherche par le radar de conduite de tir. Le temps que met le radar de conduite de tir pour réacquérir la cible dépend également directement de la taille du volume d'incertitude que définit l'image tactique. Une qualité inférieure de l'image tactique oblige le radar de conduite de tir à chercher dans un large volume. Une qualité de l'image tactique peut réduire considérablement le temps de réaction, puisque l'opération de recherche dépend d'une façon non linéaire du volume à balayer et de la probabilité de détection du radar de conduite de tir. Cette durée est soustraite du temps de réaction total disponible au décideur et nécessaire à la survie du bateau. La durée de la phase de recherche du radar de conduite de tir constitue la nouvelle mesure d'efficacité proposée dans ce document. La relation entre la qualité des pistes et la diminution du temps de réaction est définie.

Table of contents

List of figures

List of tables

1 Introduction

Modern ship-borne Command and Control (C^2) Systems should make increasing use of data fusion technology and tools. By reducing uncertainty in the existing pieces of information and providing means to infer about the missing pieces, data fusion supports the operators in compiling and analyzing the tactical picture and ultimately improving the situation awareness of the decision makers.

Data fusion has often been portrayed as a set of sequential processes that combine pieces of information in a purely passive and open loop fashion. Nevertheless, to cope with changing objectives, environments and constraints, and/or to achieve a higher performance, modern C^2 systems need many additional functions. The most essential of them is an active feedback or adaptation [1]. The introduction of such a functionality into the situation analysis process and the underlying data fusion processing defines a part of the emerging discipline of process refinement (or adaptive data fusion). However, even though it has witnessed a growing interest during the last few years, the process refinement, that represents a logical extension of the primary data fusion concepts, still lacks mature results and a shared precise definition.

The Decision Support Systems (DSS) Section, at Defence Research & Development Canada (DRDC) - Valcartier, is conducting research activities aiming at defining, developing and demonstrating advanced concepts of data fusion adaptation and sensor management that could apply to the current Command & Control Systems (CCS) of the Halifax and Iroquois Class ships as well as their possible future upgrades.

This report describes research activities that aims at bringing contributions on both theoretical and experimental sides. The objective here is to define a new Measure Of Effectiveness (MOE) that serves for the adaptation of the target tracking operation in maritime Above Water Warfare (AWW) context. Note that, in adaptive systems, Measures of Effectiveness are often required to evaluate the system behaviour with respect to high level and operational objectives. The results of such an evaluation set the low level objectives. The proposed MOE uses the search to lock-on time of the Fire Control Radar (FCR) and aims at establishing and quantifying the effect of the quality of the Maritime Tactical Picture (MTP) on the diminution of battle space size and reaction time. This work attempts to demonstrate the benefits of adaptive tracking that makes use of reaction time as a context compared to the classical open-loop tracking mode.

The Fire Control Radar is involved in the target engagement phase of the maritime AWW operations. Once a decision is made to engage a given target, the FCR is cued with positional information from the surveillance-sensor-trackers in order to re-acquire, track, and engage it. The FCR has a search time restricted by the reaction-time that depends on several factors such as the ownship weapons properties, Command & Control System (CCS) performance, the operator skill/training, and by the attacking target characteristics. Besides these factors, the time available to the decision is affected by the duration of search and lock-on operation. This time the FCR will take to re-acquire the target is directly dependent on the size of uncertainty volume defined in the tactical picture. A poor quality MTP causes the FCR to search in a large size volume. Such an MTP will have grave consequences on the battle space size, since the time for the search and lock-on depends, in

a non-linear manner, on the volume to be scanned and the probability of detection of the FCR. This time is subtracted from the total reaction time necessary to the ship survival.

This search to lock-on time constitutes the new Measure Of Effectiveness proposed in this document. A relationship between the track quality and the diminution of battle space size and reaction time is defined.

The proposed MOE can be used as an adaptation trigger in many problems in which the fire control radar is involved. A simple adaptation problem consists in turning the tracking operation on/off depending on the time available and the required quality of the tracks. A more general problem is to adapt the tracking by

1. determination of the tracking duration before cueing the FCR
2. determination of the tracking duration and nature (adapt processing and resources) to meet the requirements.

Finally, the problem of establishing relationship between the track quality and the diminution of the battle space size and the reaction time is considered. This allows defining time constraints for the search and lock-on operation, given the tactical situation. A maximum bound on the search to lock-on time is determined knowing the reaction time available to the decision maker.

This report is organized as follows. Section 2 introduces concepts and definitions of adaptive systems. This section is intended to situate the MOE computation task within the adaptation loop. Section 3 presents the fire control problem, within the whole detect-to-engage sequence. The search and lock-on problem is also described. In Section 4, the time estimation process is presented. The memorandum is concluded in Section 5. Appendix A gives the state prediction equations, while in Appendix B the coordinate conversion equations are given.

2 Adaptive systems

A system is called adaptive when it can analyze its own performance and dynamically reconfigures itself to compensate for changes in its context. The changes might be from the environment it evolves in, its objectives and/or its needs. Instead of being developed for a specific situation, adaptive systems are therefore able to handle a family of situations defined by a set of structural constraints on their context. Designers cannot foresee all the circumstances the systems will be used in. Therefore, adaptation mechanisms aim at making those systems automatically adapt to unforeseen and changing contexts, by allowing for

1. changing the system behaviour and/or performance without the need to entirely redesign it,
2. doing more on behalf of the user without a constant interaction, and
3. yielding higher performance or lower costs, or both whenever possible.

An adaptive system perceives what is going on around itself[1] and then uses this knowledge to produce appropriate actions to achieve its goals. The system must take into account the dynamic nature of the environment it interacts with to reach its goals throughout a wide range of situations with the desired level of performance[2]. Based on the newly available/inferred information, the adaptive system needs to modify its operation mode to cope with different performance criteria, variable contextual conditions and changing status of resources.

2.1 Adaptation requirements

Adaptive systems are aimed at producing applications that can readily adapt to changing operating environment and user's needs and desires. Adaptive systems need to explicitly represent the actions that can be taken and the goals that the user is trying to achieve. This makes possible for the user to change goals without the need to redesign the system. Also, the system does more on behalf of the user without constant interaction.

An adaptive system must therefore be able to measure and detect changes in its performance and respond to these changes by performing structural changes either parametric (configuration), function/reasoning-path level (processing) or at the process level (resource-bounded reasoning). See [2] for a more detailed discussion of adaptation requirements and mechanisms.

Therefore, policies that can dynamically perform these structural changes in response to a changing context are required. There are major requirements an adaptive system must meet to adequately operate in its environment. These requirements are given in Figure 1 and discussed in the followings.

[1] to obtain some idea of the state of its environment, namely, its context.
[2] which may be constant or variable.

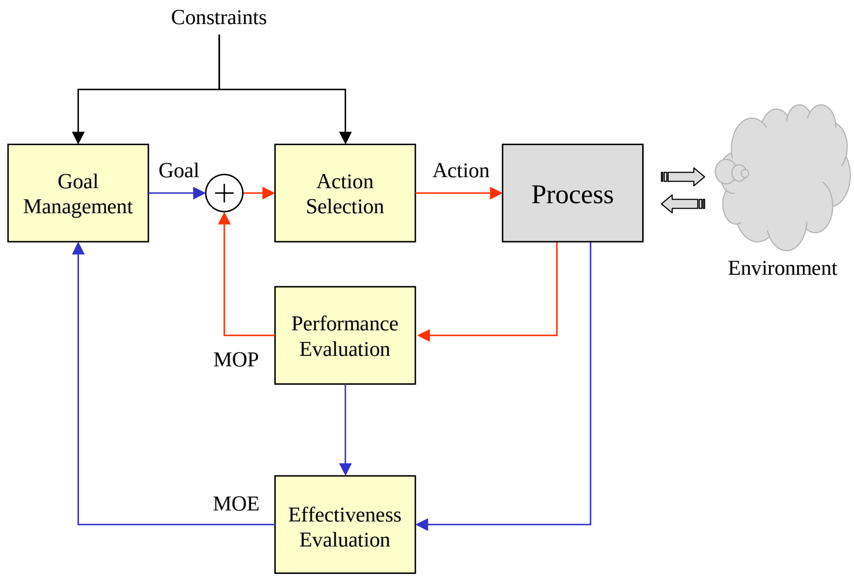


Figure 1: *Adaptation Requirements*

2.1.1 Goal management

Handling the adaptation problem presumes that the required performance of the system can be specified quantitatively to define the adaptation goals/objectives. Metrics can then be calculated and used to evaluate the system's performance (*i.e.*, the deviation from the desired behaviour). Another important requirement is to keep the adaptation effort (time and resources consumption) minimal.

Generally, specifications can be divided into two categories: performance specifications and robustness specifications. Both need to be quantified to specify the adaptation goals. Although the boundaries between the two can be fuzzy, the **performance specifications** describe the desired response of the nominal system, *i.e.*, in the absence of uncertainty. Nevertheless, for a given system, the nominal performance alone may not suffice. The system must keep providing acceptable performance even if the context is different (in some measured way) from the intended one. Therefore, **robustness specifications** are required to limit the degradation of the performance in the presence of uncertainty that may come in three basic versions:

1. uncertainties in the data (or non-parametric uncertainties),
2. uncertainties in the system models (also known as parametric uncertainties), and
3. constraints on the admissible actions (the span of the adaptation space).

It is worth mentioning that goal management is not required in all adaptive systems since adaptation objectives are often static. The desired level of performance, or reference trajectory that the adaptive system tries to achieve, is specified beforehand and remains unchanged. The difference between this reference trajectory and the measured level of performance provides a good measure of the system actual behaviour with respect the desired

one. These metrics are used as an action selection (see Subsection 2.1.4) basis to reduce any observable discrepancy.

2.1.2 Performance evaluation

Given the knowledge of the current state of the environment and the adaptation objective (*i.e.*, the context), a very important aspect of adaptive systems concerns the performance evaluation issue. This consists in finding a Measure of Performance (MOP) that, once optimized, leads to the most desirable outcome. The selected MOP serves as an action selection basis (or utility function in decision theory terminology). It is required to grade the benefits from the different possible actions so that an optimal solution can be chosen. For instance, in the target-tracking context, the information update paradigm may lead to an intuitive method of addressing the MOP selection problem. The adaptive system should therefore be able to monitor its behaviour and detect deviations from commitments or new opportunities. It should also be able to accept new needs from external sources and evaluate deviations with respect to current commitments.

2.1.3 Effectiveness evaluation

While the MOPs measure deviations from commitments on low (system) level objectives, a set of Measure of Effectiveness (MOE) is required to evaluate the system behaviour with respect to high (operational) level objectives. The results of such evaluation define the low level objectives (for the inner loop). The MOEs could depend on one or several MOPs.

The search to lock-on time we are interested in this document, which affects the battle space size and reaction time, gives a good example of a possible MOE in naval AWW context.

2.1.4 Action selection

Given the goal specifications, the environment changes and the performance and/or effectiveness measures, the adaptation problem amounts to selecting the appropriate course of actions. The objective of these actions is to maintain the discrepancy between the measured, or estimated, current performance and the desired one within some fixed bounds. This discrepancy (or error) measures how the system being considered behaves with respect to the specified level of performance. Therefore, adaptation mechanisms should be able to reason and make commitments on the goal/environment changes. Different techniques exist for action selection. The action selection defined above is the essence of the control design that consists in selecting the input (actions) to a process so that its output follows the desired specifications, while not requiring too much control effort (time/resource consumption).

When no control is used, the process input is selected at the time of design and remains the same throughout the lifetime of the process. When control is used, there are two sources for its activation. These are the goal modifications and relevant environmental changes. Each one corresponds to a different control strategy. Feedforward control is concerned with the goal changes, while feedback control handles changes in the environment. The interactions between these two components form a closely integrated loop for systems to adapt to its challenges. In the Artificial Intelligence (AI) planning context, these respectively correspond to the purely deliberative planning model and the purely reactive planning model.

Once a course of actions is selected, configuration modification functionality must offer to the adaptive system the means for its execution. The system should be able to insert the change with minimal disruption to existing behaviours. Two properties are of prime importance when executing actions. These are:

1. **Stability** – It is the most important parameter when dealing with closed-loop systems. A system is said stable if its response to a bounded input is itself bounded by a desirable range.

2. **Response time** – The response (transient) time measures the transient response of the system. That is the time the system takes to transit from one stable state to another.

3 Problem statement

In order to engage a given target, the trackers associated with the search and surveillance sensors (or tracks that result from their fusion) cue the FCR. The accuracy of the information provided to the FCR determines the volume it must scan in order to re-acquire the target and lock on it (see Figure 2). The time the FCR will take to re-acquire the target is directly dependent on the size of that volume. It may also happen that the FCR will never re-acquire the target. A poor quality MTP, *i.e.*, with large uncertainty regions, causes the FCR to search in a large size volume. Such an MTP will have grave consequences on the battle space size. The time for the search and lock-on depends, in a non-linear manner, on the volume to be scanned and the probability of detection of the FCR. This time is subtracted from the total reaction time available to the decision maker and necessary to the ship survival.

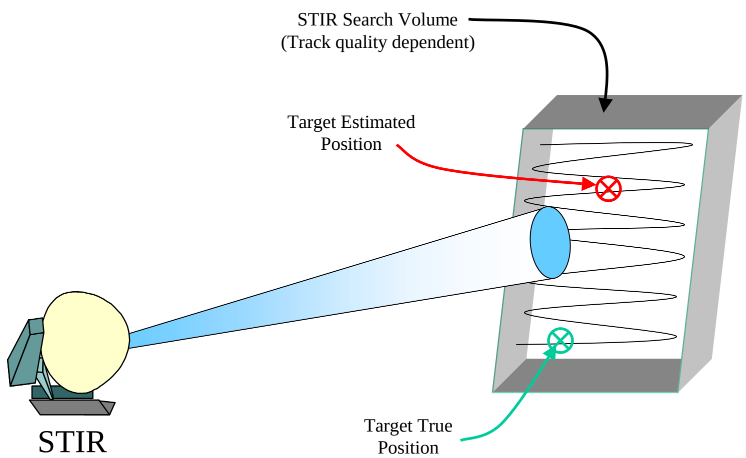


Figure 2: *FCR Search Volume*

3.1 Detect-to-engage sequence

The Fire Control Radar is involved in the target engagement[3] phase of the maritime Above Water Warfare (AWW) operations. Prior to this engagement phase there is a threat evaluation process that starts with detection/tracking and ends with the engagement decision/FCR tasking.

Upon detection by the 2D search and surveillance radars, the contact information is provided to the tracking system that maintains a more accurate estimation of the target position. Target identity inference process is executed concurrently.

In the sequel, only the target position will be considered. It is given by state estimate

$$\hat{\boldsymbol{x}} = \left[\hat{x}, \hat{y}, \hat{\dot{x}}, \hat{\dot{y}}\right]^T \qquad (1)$$

[3]It may happen that the FCR be used in the Situation Analysis Process to obtain 3D information of a given target or to make the target uncover its actual indent.

and the related covariance matrix $\boldsymbol{P}$. This information is used by subsequent processes to evaluate the threat level of the different targets present in the volume of interest.

Once an engagement decision is made, the kinematical information of the target to proceed is used to cue the FCR, *i.e.*, to delimit its search region. The FCR initiates its search within an area defined mostly by the 2D bearing-elevation information obtained from the Cartesian position $(\hat{\boldsymbol{x}}, \boldsymbol{P})$ provided by the tracking system. Following a specific pattern, the FCR will scan this region of the volume of interest until it detects and locks on the target for which a track is then maintained. The target course and speed contained in this FCR track is then used to compute a Predicted Intercept Point (PIP) inside the weapon engagement envelope.

The problem investigated here is to establish quantitatively the impact of the track quality (defined by $\boldsymbol{P}$) on the search to lock-on time of the FCR.

3.2 FCR characteristics

During the target detection phase, the search and surveillance radars scan the volume of interest for contacts. For Canadian Patrol Frigates (CPFs), search and surveillance radars (*i.e.*, SPS-49 and SG-150) work in a 2D space and therefore provide bearing (β) and range (R) information, as shown in Figure 3. No elevation is provided by 2D radar.

Note that the bearing illustrated in Figure 3 is a relative bearing rather than the true bearing. The relative bearing is the angle between the centre-line of the ship and a line pointed at the target measured clockwise from the bow. The true bearing is the angle between the true north and a line pointed at the target measured clockwise from the true north. Figure 3 also shows the Cartesian coordinate system with the X and Y axes positions.

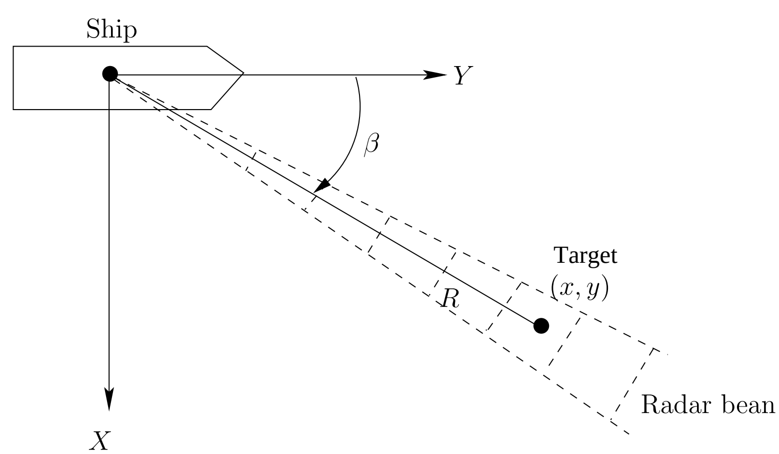


Figure 3: *Search and surveillance radar 2D scanning*

Besides the fact that it can provide the elevation (and therefore operates in 3D), the FCR (such as the CPF's STIR) differs from the search and surveillance radars by its circular and much narrower beam. This results in increased bearing and elevation precision. It also has higher transmitting frequency, so that more accurate range information is obtained.

Figure 4 shows the coordinate system for the FCR along with the range (R), the relative bearing (β) and the elevation (ε) measurements. The X and Y axes positioning is identical to 2D radars where the Y axis points at the bow of the ship. Because most of the tracking systems, either they track in 2 or 3 dimensions, work in Cartesian coordinates, the spherical measures are often converted into Cartesian ones and vice versa. This allows data to comply with the FCR and the tracking systems needs (see Appendix B for detailed conversion equations).

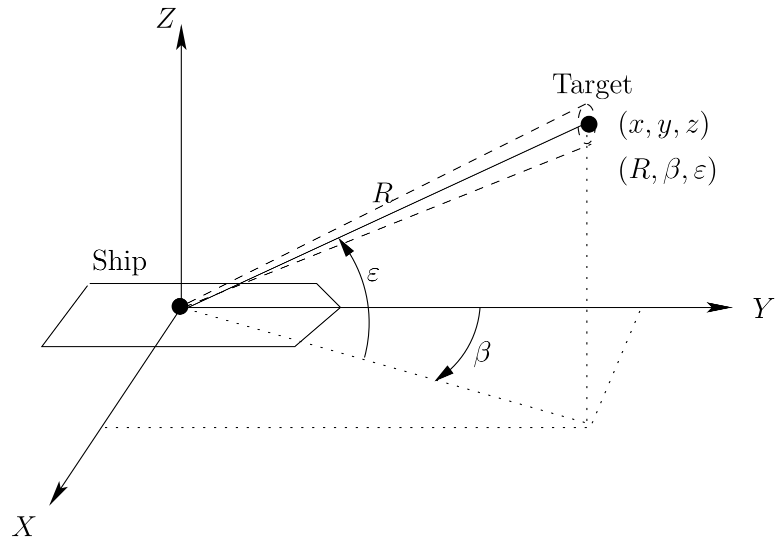


Figure 4: *Fire-control radar (FCR) scanning and coordinate system*

3.2.1 FCR parameters

The FCR is characterized by its beam dimensions. This is defined by its width b_w (or diameter b_w) and by its displacement speed $\dot{b}_\varepsilon$ in elevation and $\dot{b}_\beta$ in bearing. The FCR has a search path that can follow different patterns. This search path must be taken into account in the estimation of the search to lock-on time.

Characteristics	**Symbol**
Beam diameter	b_w
Beam speed	$\dot{b}_\varepsilon$, $\dot{b}_\beta$
Conditional detection probability	$\boldsymbol{p}_{D\|L}(x, y)$
Search path	-

Table 1: *FCR parameters*

3.3 Search to lock-on time

Once a decision is made to engage the target, the FCR is cued with positional information from the surveillance-sensor-trackers in order to re-acquire, track, and engage it. The FCR has a search time restricted by the reaction time that depends on several factors. This might be the ownship weapons properties, Command & Control System (CCS) performance, the

operator skill/training, and/or the attacking target characteristics. Given its high-risk consequences, the engagement phase requires more precise information than the threat evaluation phase. This is why the 3D Fire Control Radar takes over the less accurate 2D search and surveillance radars. To provide such accurate information, the FCR need to track the target by itself. This operation cannot be performed without a re-acquisition that involves a search and lock-on phase. This phase starts once the FCR is cued by the tracking system and begins its scan. It ends when the FCR locks on the target. The estimation of the time (d_{sl}) the FCR will take to perform this re-acquisition task defines the core of the addressed problem.

3.3.1 Applications

The new investigated MOE aims at establishing a quantitative relationship between the track quality and the search-lock-on time for the FCR. Once this MOE is fed back (see Figure 5) to the goal manager, different adaptation problems that exploit this contextual information can be defined. The output of the goal manager will provide the adaptation triggers for the inner-loop that acts on the tracking algorithms and the resource allocation in order to achieve the specified objectives. The inner-loop measures the current performance of the tracking systems, compares this performance with the specified goal, and selects appropriate actions to reduce/eliminate any observed discrepancy.

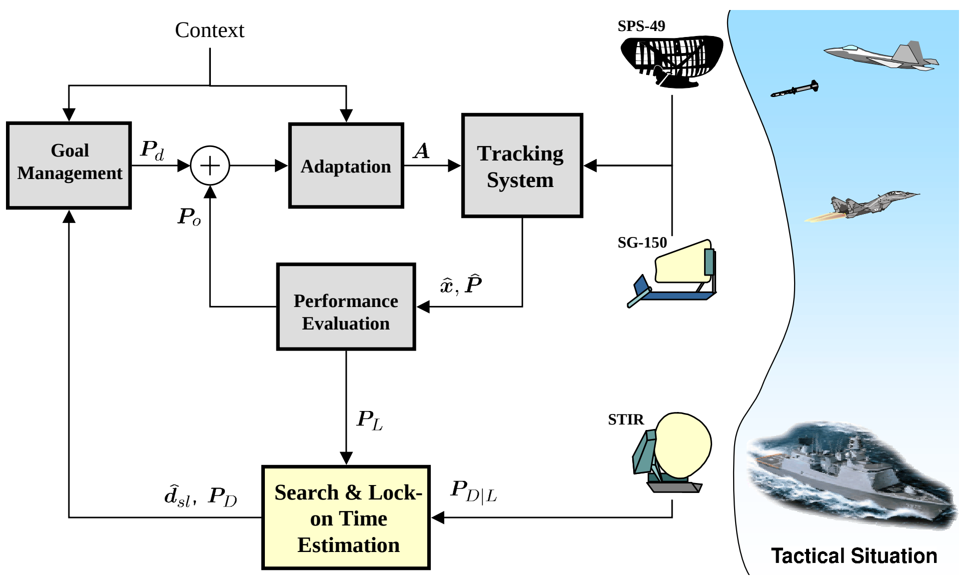


Figure 5: *FCR t_{FCR} based tracking adaptation loop*

Examples of adaptation problems currently investigated and that make use of search to lock-on time as a trigger[4], are defined below.

> **Problem 1:** Given the available reaction time, the quality of the sensor reports, the delay created by the tracking processing, and the above described MOE, one can determine whether it is more interesting to cue the FCR directly with the sensor reports or to spend more time to improve the quality of the information (through the

[4] See [2], for definitions.

tracking operation) before cueing the FCR. This adaptation problem boils down to put the tracking operation on/off depending on the context.

Problem 2: Another more general and more interesting problem consists in determining, depending on the same type information as for the first problem, the amount of time the tracking module should spend in improving the quality of the information before cueing the FCR. This problem goes beyond just switching on/off the tracking function.

Problem 3: The two problems described above assume fixed configurations. The adaptation in the first one boils down to decide whether it is more interesting to track or not. The second consists in calculating the best duration for the tracking operation. The problem described here goes further and assumes redundancy in both sensing and processing capabilities. Hence, besides the FCR, the ship has two different search and surveillance sensors. One sensor is assumed to provide more accurate contacts than the other. There is also a local tracker associated with each sensor. These trackers use a set of different tracking algorithms, each providing a different tracking performance. Now given the available reaction time, the quality of the sensor reports, the delay created by the tracking operation, and the search to lock-on time MOE, one can dynamically decide to

(a) Switch on/off the tracking (problem 1)

(b) Use one sensor for tracking with variable duration (problem 2).

(c) Use the best of the two trackers or fuse them

(d) Select among the set of available algorithms for tracking/fusion the most appropriate given the context.

(e) Combine the above listed problems in different ways.

Problem 4: This problem aims at establishing time constraints for the search and lock-on operation. This allows, given a tactical situation, calculating the maximum affordable time for search and lock-on phase. Given the reaction time available to the decision maker, the latter can determine the maximum search to lock-on time he can afford. Since this time depends on the quality of the tactical picture, the time requirements can be re-expressed as quality requirements.

It is assumed that the ownship defensive weapons $\boldsymbol{W}$ (such as SAM missiles) have an effective zone between a minimum range R_{w_-} and a maximum range R_{w_+}. The coverage zone of the FCR that guides those weapons is defined by a minimum range R_{f_-} and a maximum range R_{f_+} (see Figure 6).

Let $\dot{R}_w$ be the defensive weapon speed and t_{w_-} the time this weapon takes before entering its effective zone (at range R_{w_-}). Then

$$t_{w_-} = \frac{R_{w_-}}{\dot{R}_w} \tag{2}$$

Given t_{w_-} and R_{w_-}, there is a minimum distance R_{t_-} from the ship (see Equation 3) where the threat $\boldsymbol{R}$ must be detected by the FCR and engaged by $\boldsymbol{W}$.

$$R_{t_-} = R_{w_-} - \dot{R}_w t_{w_-} \tag{3}$$

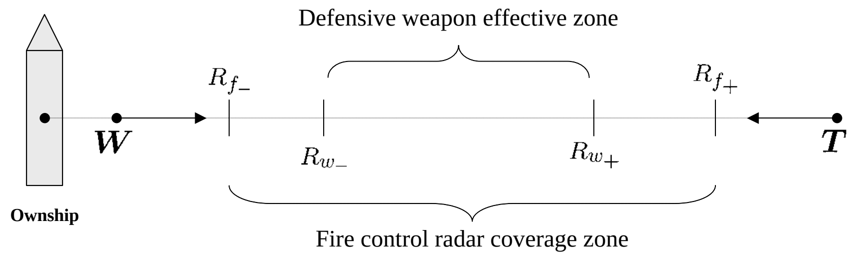


Figure 6: *Scenario characteristics*

where $\dot{R}_w < 0$. If the defensive weapon $\boldsymbol{W}$ is launched while the threat $\boldsymbol{T}$ has already passed R_{t_-}, it will be too late. The defensive weapon will not reach its target, since the latter will be outside its effective zone. Therefore, the weapon $\boldsymbol{W}$ must be fired while the threat $\boldsymbol{R}$ is beyond R_{t_-}. The time available to the FCR (t_{f_a}) is given by

$$t_{f_a} = t_{w_-} - \left[\frac{R_{t_c} - R_{w_-}}{\dot{R}_t}\right] \tag{4}$$

where $\dot{R}_t < 0$ and R_{t_c} is the range of the threat when the FCR is cued. The validity interval of this time is given by

$$R_{f_+} \geq R_{t_c} \geq \max\{R_{f_-}, R_{w_-} + d_{sl}\dot{R}_t\} \tag{5}$$

where d_{sl} is the duration of the search and lock-on operation of the FCR. The duration is limited by the available time t_{f_a} given by Equation 4. Obviously, the available time is maximum when the FCR is cued at R_{f_+}

$$t_{f_a} = t_{w_-} - \left[\frac{R_{f_+} - R_{w_-}}{\dot{R}_t}\right] \tag{6}$$

The search and lock-on duration d_{sl} represents the core problem discussed in this document.

One can easily see that several adaptation problems, that exploit the high level contextual information provided by the search and lock on time, can be defined and used to improve the quality of the response of the ownship to a given threat.

4 Search to lock-on time MOE computation

The estimation of the duration of the search and lock operation (d_{sl}) may vary depending on several factors. More precisely, and besides $\hat{\boldsymbol{x}}$ and $\boldsymbol{P}$, this duration estimate $\hat{d}_{sl}$ will depend on

1. the FCR characteristics, such as the beam dimensions, displacement speed, and the search pattern,

2. The conditional detection probability $\boldsymbol{p}_{D|L}$ which is also a characteristic of the FCR. This measures the probability that the FCR detects the target, given that the latter is within its gate.

3. The localization probability distribution $\boldsymbol{p}_L$, which is defined by the 2D track information provided by the tracking system.

4. The motion prediction model. Since the FCR is searching for non-stationary target, the 2D track information must be time-updated using a dynamic model of the target motion.

5. Some empirical parameters (defined in the following sections).

All these factors must be known or determined in order to estimate the expected duration of the search and lock-on operation. Within this process estimation, the possibility of sweeping all the search area without finding the target must also be considered. In this case, the search must be restarted within a redefined area wider than the previous one. The area redefinition must take into consideration the possibility that the target could have moved meanwhile and could be outside of the previously defined initial search area. Several iterations may be required before detecting the target. In short, the problem we are investigating addresses the following issues

1. Delimiting the search area for the FCR based on the information provided by the tracking system associated with search radars.

2. Determining the detection and localization probabilities based on covariance information provided by the same tracking system.

3. Defining a method for the estimation of the search and lock duration based on the above-given information and the characteristics of the underlying FRC.

4. Updating periodically both the search area and localization probabilities. This is required to take into account any possible movement of the target during the search operations. This boils down to updating the target state $\hat{\boldsymbol{x}}$ and the covariance matrix $\hat{\mathbf{P}}$ and perform tasks 1 & 2 on the newly obtained information.

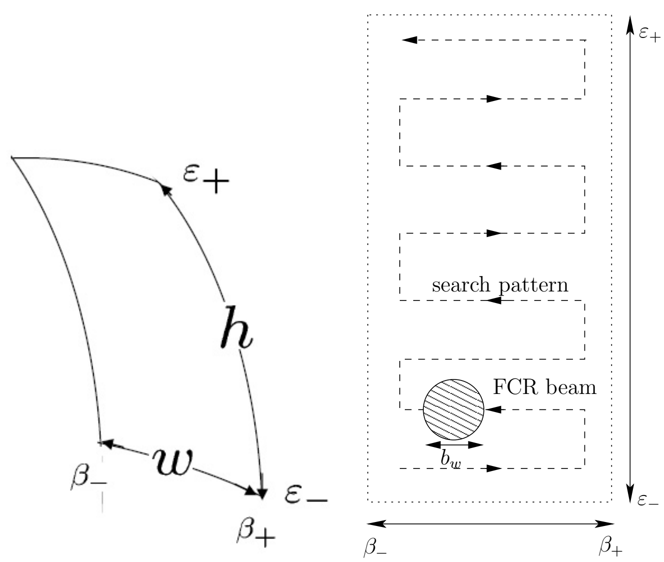
Each of these issues raises several sub-problems that will be discussed in the next sections.

4.1 FCR search area

Since the tracking system associated with the search radars provides a state $\hat{\boldsymbol{x}}$ and the covariance matrix $\boldsymbol{P}$, this information is converted into probability assuming normal distributions. The search area $\boldsymbol{A}$ is determined according to the target distribution in bearing β along with a confidence level α. Since the 2D tracking system provides no elevation information, the target localization probability is unknown in elevation (ε). A uniform probability distribution is then considered with an elevation range $[0°, 90°]$. Note that some identification/classification information may be used to reduce this range based on the threat characteristics (*e.g.*, high diver, sea-skimmer, etc..).

The search area $\boldsymbol{A}$ is modeled such that the area is partitioned into squared cells having dimensions equal to the radar beam dimensions $b_w \times b_w$. Each cell (i, j) has its associated localization probability of the target $\boldsymbol{p}_L(i, j)$. This localization probability is defined according to the track information provided to the FCR by the tracking system.

The search area is therefore bounded in bearing according to a confidence level α, which determines the minimum bearing β_- and the maximum bearing β_+ in which to search. The bounds ε_- and ε_+ in elevation are determined by some predefined criteria or limitations. Ideally, $\boldsymbol{A}$ has to be bounded in bearing so that the locations with a very low localization probability $\boldsymbol{p}_L$ are kept outside the search area $\boldsymbol{A}$. Figure 7 shows an example of an area scanned by the FCR.



(a) Area in the 3D space (b) Bearing/elevation plane

Figure 7: *Area scanned by the FCR*

Let $N(\hat{\beta}, \hat{\sigma}_\beta)$ be the bearing normal distribution (see Section 4.2.1). $N(\hat{\beta}, \hat{\sigma}_\beta)$ can be con-

verted into a standard normal distribution $N(0,1)$ (z scores) using

$$z = \frac{\beta - \hat{\beta}}{\hat{\sigma}_\beta} \tag{7}$$

From the confidence level α, one must find a lower and upper bounds that cut out 1-α of the area under the curve. Looking up in a z table, those bounds correspond to the z scores $-z_{(\frac{1-\alpha}{2})}$ and $z_{(\frac{1-\alpha}{2})}$ (see Figure 8). β_- and β_+ are defined according to $-z_{(\frac{1-\alpha}{2})}$ and $z_{(\frac{1-\alpha}{2})}$, as follows

$$\beta_- = \hat{\beta} - z_{(\frac{1-\alpha}{2})}\hat{\sigma}_\beta \tag{8}$$

$$\beta_+ = \hat{\beta} + z_{(\frac{1-\alpha}{2})}\hat{\sigma}_\beta \tag{9}$$

The width w and the height h of $\boldsymbol{A}$ are then defined by

$$w = \beta_+ - \beta_- = 2z_{(\frac{1-\alpha}{2})}\hat{\sigma_\beta} \tag{10}$$

$$h = \varepsilon_+ - \varepsilon_- \tag{11}$$

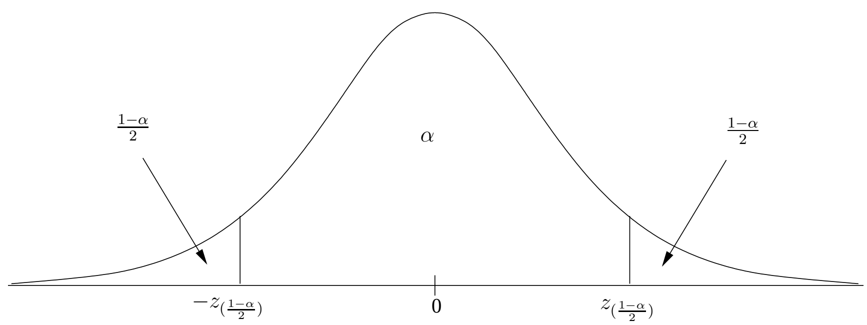


Figure 8: *Bounds in the standard normal distribution with a confidence level* α

4.2 Detection and localization probabilities

To each cell (i,j) of the search area $\boldsymbol{A}$ can then be associated a detection probability $\boldsymbol{p}_D(i,j)$. The detection probability $\boldsymbol{p}_D$ is the probability that the FCR detects the target when aiming at a particular location (cell). This probability is defined as follows:

$$\boldsymbol{p}_D = \boldsymbol{p}_L \times \boldsymbol{p}_{D|L} \tag{12}$$

where $\boldsymbol{p}_L$ is the localization probability and $\boldsymbol{p}_{D|L}$ the conditional detection probability of the used sensor. The probability $\boldsymbol{p}_L(i,j)$ depends on the target distribution. The localization probabilities $\boldsymbol{p}_L$ are normalized, so that their summation in $\boldsymbol{A}$ amounts to 1.0. The probability $\boldsymbol{p}_{D|L}$ depends on the properties and performance of the used sensor (*i.e.*, the FCR). This is a conditional detection probability that gives the probability of detecting the target, assuming its location is known ($\boldsymbol{p}_L = 1.0$).

4.2.1 Target localization probability

As mentioned previously, the localization probability $\boldsymbol{p}_L$ at a particular position is given using the probability density function of the target localization. Let us suppose a Gaussian target distribution in bearing β and a uniform distribution in elevation ε. The probability density function $\Phi(\beta)$ of the target is univariate and given by the normal distribution $N(\hat{\beta}, \hat{\sigma}_\beta)$

$$\Phi(\beta) = N(\hat{\beta}, \hat{\sigma}_\beta) = \frac{1}{\sqrt{2\pi\hat{\sigma}_\beta}} \exp\left[-\frac{(\beta-\hat{\beta})^2}{2\hat{\sigma}_\beta^2}\right] \tag{13}$$

In practice, the support of the probability density function is finite and must be bounded.

4.2.2 Bounded-support probability

The bounded support and normalized probability density function Φ_b is given by

$$\Phi_b(\beta) = \begin{cases} \dfrac{\Phi(\beta)}{\sum_\beta \Phi(\beta)}, & \beta_- < \beta < \beta_+ \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Hence, the localization probability distribution can be represented by the new defined probability density function

$$\boldsymbol{p}_L = \Phi_b \tag{15}$$

4.2.3 Calculating $\Phi(\beta)$ from $\Phi(\mathbf{x})$

In Subsection 4.2.1, $\hat{\sigma}_\beta$ was required to calculate $\Phi(\beta)$. Appendix B gives all the required equations to calculate the different conversion of Cartesian information to polar/spherical information and vice versa. Also is given the equations that allow for computing the associated variances.

4.3 Average search time estimation

In this section, the expected duration of the search and lock task is estimated. First, the case of single-area search is considered in Subsection 4.3.1. The case of recursive, multi-area search is considered in Subsection 4.3.2

4.3.1 One area estimation

Here the average time to find the target in a single search area $\boldsymbol{A}$ is estimated. The general case will first be described, then the specific example of a standard Fixed Swath search path with a rectangular search area is presented.

4.3.1.1 General case

The average search time $\bar{t}$ in an area $\boldsymbol{A}$ is calculated by summing the detection probability for each cell (i, j) multiplied by the elapsed time $t_{FCR}(i, j)$ when the beam of the FCR

goes over cell (i, j). The elapsed time depends on the beam displacement speed and on its pathway.

$$\bar{t} = \sum_i \sum_j \boldsymbol{p}_D(i,j) \times t_{FCR}(i,j) \tag{16}$$

Note that the position of a cell (i, j) is given in the Spherical coordinates by

$$(\beta, \varepsilon) = (\beta_- + ib_w + \frac{b_w}{2}, \varepsilon_- + jb_w + \frac{b_w}{2}) \tag{17}$$

4.3.1.2 Fixed swath path

Lets consider the particular example of the standard Fixed Swath search path, shown in Figure 7. The area to be searched is bounded as described in Section 4.2. The average time $\bar{t}$ of the standard Fixed Swath search path is calculated by adding the average time for the displacements in bearing and elevation, as follows

$$\bar{t} = \bar{t}_\beta + \bar{t}_\varepsilon \tag{18}$$

with $\bar{t}_\beta$ and $\bar{t}_\varepsilon$ being the average time spent displacing along in bearing and elevation respectively. Time $\bar{t}_\varepsilon$ is calculated according to

$$\bar{t}_\varepsilon = \sum_{j=0}^{\frac{h}{b_w}-1} \frac{b_w}{\dot{b}_\varepsilon} \sum_{k=0}^{j} \left[1 - \prod_{i=0}^{\frac{w}{b_w}-1} \boldsymbol{p}_D(i,k)\right] \tag{19}$$

where $b_w/\dot{b}_\varepsilon$ is the beam displacement time of a b_w angle in elevation. The time $\bar{t}_\beta$ is calculated by summing the average time for the left to right sweeps ($\bar{t}_{lr}$) and the average time for the right to left sweeps ($\bar{t}_{rl}$)

$$\bar{t}_\beta = \bar{t}_{lr} + \bar{t}_{rl} \tag{20}$$

with

$$\bar{t}_{lr} = \sum_{j=0}^{j \le \frac{h-b_w}{2b_w}} \sum_{i=0}^{\frac{w}{b_w}-1} \boldsymbol{p}_D(i,2j) \frac{b_w}{\dot{b}_\beta} \left[\frac{w}{b_w} 2j + i\right] \tag{21}$$

where

$$\frac{b_w}{\dot{b}_\beta} \left[\frac{w}{b_w} k + i\right] \tag{22}$$

is the cumulated time according to cell (i, k), and with

$$\bar{t}_{rl} = \sum_{j=0}^{j \le \frac{h-b_w}{2b_w}} \sum_{i=0}^{\frac{w}{b_w}-1} \boldsymbol{p}_D(i,2j+1) \frac{b_w}{\dot{b}_\beta} \left[\frac{w}{b_w}(2j+1) + (\frac{w}{b_w} - 1) - i\right] \tag{23}$$

where the cumulated time according to cell (i, k) is

$$\frac{b_w}{\dot{b}_\beta} \left[\frac{w}{b_w} k + (\frac{w}{b_w} - 1) - i\right] \tag{24}$$

4.3.1.3 Gaussian $\boldsymbol{P_L}$

Having a Gaussian localization probability distributions, the average search time is related to the target estimated bearing position $\hat{\beta}$ in the search area. The average time spent searching the target will be the time the FCR takes up to the estimated position in the bearing. With Gaussian distributions, the target estimated bearing position ($\hat{\beta}$) is at the center of the search area. Then, the sweeping time up to the estimated position is half the time to sweep the area totally. Let T be the time to sweep the area totally

$$T = \frac{(w - b_w)}{\dot{b}_\beta}\frac{h}{b_w} + \frac{h - b_w}{\dot{b}_\varepsilon} \tag{25}$$

where

$$\frac{(w - b_w)}{\dot{b}_\beta}\frac{h}{b_w}$$

is the time spent displacing along the bearing axis, while

$$\frac{h - b_w}{\dot{b}_\varepsilon}$$

is the time spent displacing along the elevation axis. Thus, the average search time in one area with a Gaussian localization probability distribution is

$$\bar{t} = \frac{T}{2} = \frac{(w - b_w)}{2\dot{b}_\beta}\frac{h}{b_w} + \frac{h - b_w}{2\dot{b}_\varepsilon} \tag{26}$$

4.3.2 Recursive estimation

The track information $\hat{\boldsymbol{x}}$ and $\boldsymbol{P}$ provided to the FCR by the tracking system is used to define the initial search area $\boldsymbol{A}$ and the associated localization probability Φ_b. These two quantities allow for computing the average time ($\bar{t}$) required to re-acquire the target by the FCR, assuming that the probability to find the target in $\boldsymbol{A}$ is equal to 1.0.

Lets $\boldsymbol{A}_n$ be the area the FCR has to sweep at the n^{th} scan. This means that the FCR was unsuccessful in re-acquiring the target in its $n-1$ previous scans. The time $\bar{t}_n$ defined in Equation 16 is then the average search time for scan n only. T_n is the time required to scan area $\boldsymbol{A}_n$ totally, as defined by Equation 25. At scan n, if one is sure the target will be detected, the required search time would be at least

$$T_{n-1} + T_{n-2} + ... + T_1$$

and, at most,

$$T_n + T_{n-1} + T_{n-2} + ... + T_1$$

The expected time would be

$$\bar{t}_n + T_{n-1} + T_{n-2} + ... + T_1$$

When the FCR is cued, one is able to calculate T_1, the time to scan the initial area entirely according to $\hat{\boldsymbol{x}}_1$ and $\boldsymbol{P}_1$, the state estimate and its corresponding error covariance matrix

just before the FCR begins scanning. T_1, $\hat{\boldsymbol{x}}_1$ and $\boldsymbol{P}_1$ then allow for computing $\hat{\boldsymbol{x}}_2$ and $\boldsymbol{P}_2$, the state estimate and its corresponding error covariance matrix just before the FCR begins the second scan, using the state prediction model (Appendix A). Recursively, $\hat{\boldsymbol{x}}_2$ and $\boldsymbol{P}_2$ will engender T_2 and $\bar{t}_2$, and so on. Now, let

1. $\boldsymbol{P}_{D_n}$ be the probability to detect the target at scan n in a search area $\boldsymbol{A}_n$, as given by Equation 27,

2. $\boldsymbol{P}_{S_n}$ be the probability that the search goes through n trials before finding the target, as defined in Equation 28, and

3. $\hat{t}_{S_n}$ be the estimated elapsed time going up to n trials to find the target, as given by Equation 29.

Table 2 shows the estimated search times and the probabilities of going through some number of trials before finding the target.

# Scans	Estimated time	Probability
1	$\hat{t}_{S_1} = \bar{t}_1$	$\boldsymbol{P}_{S_1} = \boldsymbol{P}_{D_1}$
2	$\hat{t}_{S_2} = (T_1 + \bar{t}_2)$	$\boldsymbol{P}_{S_2} = (1 - \boldsymbol{P}_{D_1})\boldsymbol{P}_{D_2}$
3	$\hat{t}_{S_3} = (T_1 + T_2 + \bar{t}_3)$	$\boldsymbol{P}_{S_3} = (1 - \boldsymbol{P}_{D_1})(1 - \boldsymbol{P}_{D_2})\boldsymbol{P}_{D_3}$
4	$\hat{t}_{S_4} = (T_1 + T_2 + T_3 + \bar{t}_4)$	$\boldsymbol{P}_{S_4} = (1 - \boldsymbol{P}_{D_1})(1 - \boldsymbol{P}_{D_2})(1 - \boldsymbol{P}_{D_3})\boldsymbol{P}_{D_4}$
$\vdots$	$\vdots$	$\vdots$

Table 2: *Estimated time and probability related to the number of trials*

$$\boldsymbol{P}_{D_n} = 1 - \prod_{(i,j)\in \boldsymbol{A}_n} \Big[1 - \boldsymbol{p}_D(i,j)\Big] \tag{27}$$

$$P_{S_n} = \boldsymbol{P}_{D_n} \cdot \prod_{k=1}^{n-1} (1 - \boldsymbol{P}_{D_k}) \tag{28}$$

$$\hat{t}_{S_n} = \bar{t}_n + \sum_{k=1}^{n-1} T_k \tag{29}$$

where $\prod_{(i,j)\in \boldsymbol{A}_n}$ stands for "product on all cells in $\boldsymbol{A}_n$". Adding the probabilities $\boldsymbol{P}_{S_n}$ for each event n (number of trials) multiplied by the related time spent $\hat{t}_{S_n}$, one obtains the expected search time $\hat{d_{sl}}$ for a target considering N trials

$$\hat{d_{sl}} = \sum_{n=1}^{N} \hat{t}_{S_n} \boldsymbol{P}_{S_n} \tag{30}$$

Remark

To obtain a plausible estimation from Equation 30, a number of iterations N must be predetermined or bounded beforehand. To avoid that probability detection $\boldsymbol{P}_D$ vanishes, the re-acquisition should happen within few iterations. Otherwise, the search areas will

grow unboundedly and become too large (*i.e.*, the localization probability, too low) to detect any object.

Although $\hat{t}_{S_n}$ (and the search area $\boldsymbol{A}$) grows exponentially and can become intractable very fast (exponentially) for some particular initial parameters, $\boldsymbol{P}_{S_n}$ will vanish rapidly, so that the solution should converge as the number of trial increases. The periods of time corresponding to the first trials should be the most contributive to the solution.

4.3.3 Stop conditions

As mentioned previously, the process of search and lock-on may involve many sweeps in order to re-acquire the target and lock on it. The process could even go non-stop and limitless if the FCR never re-acquires the target. For the purpose of time estimation, the process must be stopped at some instant that can be defined as a specific number of sweeps (or iterations). The problem then comes to find how the number of sweeps will be set. The solution will depend on the application, such as those covered with the problems mentioned in Subsection 3.3.1.

However, besides the application related issues, other factors may set some limits to the number of sweeps. One factor is the workload involved. In the search process, the search area grows exponentially at each trial and may become very large after some number of sweeps. It is possible that the search area becomes so wide that the estimated search to lock-on time begins to grow with the number of iterations considered instead of stabilizing to a specific value. This may lead to an infinite estimated search to lock-on time.

In such a situation, $\hat{t}_{S_n}$ is so large that, even though $\boldsymbol{P}_{S_n}$ becomes smaller as the number of iterations (n) increases, the product $\hat{t}_{S_n}\boldsymbol{P}_{S_n}$ (in Equation 30) increases instead of decreasing towards zero. When such a situation is detected, the iterative estimation should be stopped to avoid unnecessary calculus.

As an example of a stop condition, one can avoid excessive calculations by setting the total detection probability required $\boldsymbol{P}_{D_{tot}}$. The total detection probability $\boldsymbol{P}_{D_{tot}}$ is related to the number of trials N considered (Equation 31). Having $\boldsymbol{P}_{D_{tot}} = 1$ would require an infinite number of trials. Thus, by setting $\boldsymbol{P}_{D_{tot}} < 1$, one gets a finite number of trials N

$$\boldsymbol{P}_{D_{tot}} = 1 - \boldsymbol{P}_{m_{tot}} = 1 - \prod_{n}^{N} \boldsymbol{P}_{m_n} = 1 - \prod_{n}^{N} \left[1 - \prod_{(i,j)\in \boldsymbol{A}_n} (1 - \boldsymbol{p}_D(i,j)) \right] \tag{31}$$

where $\boldsymbol{P}_{m_{tot}}$ is the total miss probability and $\boldsymbol{P}_{m_n}$ is the miss probability at trial n. Note that $\boldsymbol{p}_D(i,j)$ is not the same in each trial n. In practice, Equation 31 is calculated iteratively by considering each trial n, and thereby the iteration should stop when $\boldsymbol{P}_{D_{tot}}$ reaches a specific value.

5 Conclusion

The Decision Support Systems (DSS) Section at Defence Research & Development Canada (DRDC) – Valcartier is conducting research activities aiming at defining, developing, and demonstrating advanced data fusion adaptation concepts. The objective of the reported work, as part of these research activities, is to propose a new Measure Of Effectiveness (MOE) based on the search to lock-on time of the Fire Control Radar (FCR). This proposed Measure Of Effectiveness aims at providing the required adaptation context of the target tracking process in naval Above Water Warfare. This MOE establishes a quantitative relationship between the maritime tactical picture quality and the search to lock-on time of the FCR.

A method to estimate the expected duration of the search and lock-on operation has been developed. This estimation takes into consideration the FCR search pattern, the beam width, and the displacement speed. It also considers the localization and detection probabilities. The method iterates over a series of sweeps the FCR can go through before re-acquiring and locking on the target. These sweeps must be limited following specific conditions such as the total detection probability cumulated after each sweep.

Within the framework of the adaptive tracking system of the FCR, the proposed MOE being used tackles and illustrates different adaptation problems. This represents a first step towards building a tracking system that meets all the adaptive requirements.

References

[1] Roy, J. and Bossé, É (1995), Definition of a Performance Evaluation Methology for Sensor Data Fusion Systems. DREV TR 9423. UNCLASSIFIED.

[2] Benaskeur, Abder Rezak and Bossé, Éloi (2003), Adaptive Data Fusion Concepts for Tracking in a Tactical Situation, In *RTO - SET 059 Symposium on Target Tracking & Data Fusion for Military Observation Systems*.

Annex A: Prediction model

The tracked target is assumed to be moving in a 2D space, where the acceleration acts as an input[5]. Cartesian coordinates are used in order to be in a linear system. The state to be estimated is composed of the target's coordinates, *viz.*, the position and the linear velocity. With the following state variable notation

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{v} = \begin{bmatrix} \boldsymbol{v}_1 \\ \boldsymbol{v}_2 \end{bmatrix} \tag{A.1}$$

the equations of such a target can be expressed as

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_k \boldsymbol{x}_k + \boldsymbol{\Gamma} \boldsymbol{v}_k \tag{A.2}$$

where $\boldsymbol{v}$ is vector of random variables that reflects the unforeseeable variation of the acceleration in both directions and the state transition matrix is given by

$$\boldsymbol{F}_k = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.3}$$

where Δt is the time increment. The matrix $\boldsymbol{\Gamma}$ in (A.2) depends on the model used to represent the discrete-time nature of the process noise $\boldsymbol{v}$. Two examples are given below. The first one

$$\boldsymbol{\Gamma} = \begin{bmatrix} \Delta t^2/2 & 0 \\ 0 & \Delta t^2/2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \tag{A.4}$$

represents to the pulse model, while the following one

$$\boldsymbol{\Gamma} = \begin{bmatrix} \dfrac{\sqrt[3]{\Delta t^2}}{2\sqrt{3}} & \dfrac{\sqrt[3]{\Delta t^2}}{2} & 0 & 0 \\ 0 & 0 & \dfrac{\sqrt[3]{\Delta t^2}}{2\sqrt{3}} & \dfrac{\sqrt[3]{\Delta t^2}}{2} \\ 0 & \sqrt{\Delta t} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\Delta t} \end{bmatrix} \tag{A.5}$$

is used by the Brownian motion modeling. Note that the latter requires four random variables instead of two. If the process noises are assumed to have the same standard

[5] Since the acceleration may change unforeseeably, it is often modeled as a random variable, and so will be as such.

deviation $\boldsymbol{\delta}_v$, the pulse model results in the following process noise covariance matrix

$$\boldsymbol{Q} = \boldsymbol{\delta}_v^2 \boldsymbol{\Gamma}\boldsymbol{\Gamma}^T \tag{A.6}$$

$$= \boldsymbol{\delta}_v^2 \begin{bmatrix} \frac{\Delta t^4}{4} & 0 & \frac{\Delta t^3}{2} & 0 \\ 0 & \frac{\Delta t^4}{4} & 0 & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & 0 & \Delta t^2 & 0 \\ 0 & \frac{\Delta t^3}{2} & 0 & \Delta t^2 \end{bmatrix} \tag{A.7}$$

while the covariance matrix for the brownian motion is given by

$$\boldsymbol{Q} = \boldsymbol{\delta}_v^2 \boldsymbol{\Gamma}\boldsymbol{\Gamma}^T \tag{A.8}$$

$$= \boldsymbol{\delta}_v^2 \begin{bmatrix} \frac{\Delta t^3}{3} & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^3}{3} & 0 & \frac{\Delta t^2}{2} \\ \frac{\Delta t^2}{2} & 0 & \Delta t & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 & \Delta t \end{bmatrix} \tag{A.9}$$

We assume that a Kalman filter is used for tracking. Since there is no sensor report during the search and lock-on phase, only the time update part of the filter is considered. The state dynamic model (Equation A.2) serves for the prediction of the state variable $\hat{\boldsymbol{x}}$ and the corresponding covariance matrix $\boldsymbol{P}_{n+1}$ at trial $n+1$ from their values at trial n and T_n. Note that T_n is the time to sweep the area A_n totally at scan n (Equation 25).

1. **Prediction of the state** — The most recent available *a posteriori* estimate of the state, namely $\hat{\boldsymbol{x}}_{T_n|T_n}$, is injected in the system model to yield the new *a priori* estimate $\hat{\boldsymbol{x}}_{k+1|k}$ of the state at the time instant T_{n+1}.

$$\hat{\boldsymbol{x}}_{T_{n+1}|T_n} = \boldsymbol{F}_{T_n} \hat{\boldsymbol{x}}_{T_n|T_n} \tag{A.10}$$

2. **Prediction of the covariance matrix** — A prediction of the estimation error covariance matrix is performed, at the time instance T_{n+1}. Its computation is based upon the most recent available *a posteriori* value $\boldsymbol{P}_{T_n|T_n}$.

$$\boldsymbol{P}_{T_{n+1}|T_n} = \boldsymbol{F}_{T_n} \boldsymbol{P}_{T_n|T_n} \boldsymbol{F}_{T_n}^T + \boldsymbol{Q}^T$$

Note that the system noise $\boldsymbol{v}$ is not used in the state prediction equation (A.10). This is due to the fact that the best prediction one can make for a white noise is its mean, which happens to be equal to zero here.

Annex B: Coordinate systems

The spherical coordinate system family allows for tracking in the same coordinate system as the sensor reports. It is particularly more suitable when the sensor reports only partial information (*e.g.*, angle only). For the widely used constant velocity and constant acceleration models of the target flight trajectories, tracking in the spherical system makes the motion equations lead to highly nonlinear systems. In the following, when required, the spherical coordinate system (R, θ, ϕ) is replaced by a slightly modified version, hereafter simply referred to as the spherical coordinate system (R, β, ϵ). R is a distance coordinate that corresponds to the slant range relative to the interceptor, β (bearing) is defined, in the horizontal plan, as the angle relative to the North, while ϵ (elevation) is defined as the elevation relative to the Horizon (see Figure B.1). This modification is only introduced to conserve the compatibility with the previous works.

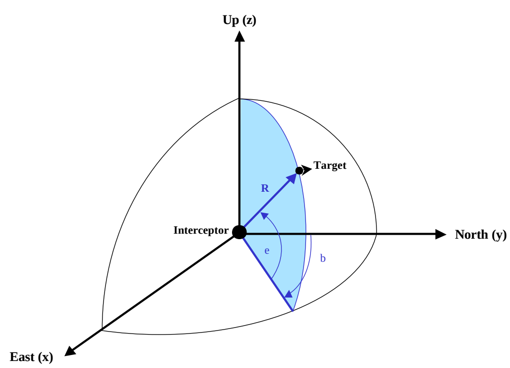


Figure B.1: *Spherical coordinate system (adapted convention)*

The Cartesian coordinate system, on the other hand, represents the natural choice for the widely used constant velocity and constant acceleration dynamical models of the target flight trajectories. The main advantage of this system lies in that it allows for the use of the well-known standard Kalman filtering technique, which is applicable only when the state transition and observation equations are expressed by linear functions.

B.1 Cartesian 3D to spherical conversion

Based on the available Cartesian information, the spherical coordinates and their derivatives are expressed as

$$R = (x^2 + y^2 + z^2)^{1/2} \tag{B.1}$$
$$\dot{R} = (x\dot{x} + y\dot{y} + z\dot{z})(x^2 + y^2 + z^2)^{-1/2} \tag{B.2}$$
$$\beta = \tan^{-1} xy^{-1} \tag{B.3}$$
$$\dot{\beta} = (y\dot{x} - x\dot{y})(x^2 + y^2)^{-1} \tag{B.4}$$
$$\epsilon = \tan^{-1} z(x^2 + y^2)^{-1/2} \tag{B.5}$$
$$\dot{\epsilon} = (x^2 + y^2)^{-1/2}\left[\dot{z} - z(x\dot{x} + y\dot{y} + z\dot{z})(x^2 + y^2 + z^2)^{-1}\right] \tag{B.6}$$

Each of the above-given variables can be rewritten as

$$\xi = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} & \frac{\partial \xi}{\partial z} & \frac{\partial \xi}{\partial \dot{x}} & \frac{\partial \xi}{\partial \dot{y}} & \frac{\partial \xi}{\partial \dot{z}} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \triangleq \mathcal{J}_\xi^T \boldsymbol{X} \tag{B.7}$$

The corresponding error variances/covariances are given by

$$\begin{aligned}
\sigma_{\xi\zeta}^2 &= E\left[\xi\zeta^T\right] = \mathcal{J}_\xi^T E\left[\boldsymbol{X}\boldsymbol{X}^T\right]\mathcal{J}_\zeta \\
&= \mathcal{J}_\xi^T \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & \sigma_{xz}^2 & \sigma_{x\dot{x}}^2 & \sigma_{x\dot{y}}^2 & \sigma_{x\dot{z}}^2 \\ & \sigma_y^2 & \sigma_{yz}^2 & \sigma_{y\dot{x}}^2 & \sigma_{y\dot{y}}^2 & \sigma_{y\dot{z}}^2 \\ & & \sigma_z^2 & \sigma_{z\dot{x}}^2 & \sigma_{z\dot{y}}^2 & \sigma_{z\dot{z}}^2 \\ & & & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}}^2 & \sigma_{\dot{x}\dot{z}}^2 \\ & & & & \sigma_{\dot{y}}^2 & \sigma_{\dot{y}\dot{z}}^2 \\ & & & & & \sigma_{\dot{z}}^2 \end{bmatrix} \mathcal{J}_\zeta
\end{aligned}$$

which can be rewritten, using the partial derivatives, as

$$\begin{aligned}
\sigma_{\xi\zeta}^2 = {} & \frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial x}\sigma_x^2 + \frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial y}\sigma_y^2 + \frac{\partial \xi}{\partial z}\frac{\partial \zeta}{\partial z}\sigma_z^2 + \frac{\partial \xi}{\partial \dot{x}}\frac{\partial \zeta}{\partial \dot{x}}\sigma_{\dot{x}}^2 + \frac{\partial \xi}{\partial \dot{y}}\frac{\partial \zeta}{\partial \dot{y}}\sigma_{\dot{y}}^2 + \frac{\partial \xi}{\partial \dot{z}}\frac{\partial \zeta}{\partial \dot{z}}\sigma_{\dot{z}}^2 \\
& + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial y} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial y}\right]\sigma_{xy}^2 + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial z} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial z}\right]\sigma_{xz}^2 + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial \dot{x}} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial \dot{x}}\right]\sigma_{x\dot{x}}^2 \\
& + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{x\dot{y}}^2 + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial \dot{z}} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial \dot{z}}\right]\sigma_{x\dot{z}}^2 + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial z} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial z}\right]\sigma_{yz}^2 \\
& + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial \dot{x}} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial \dot{x}}\right]\sigma_{y\dot{x}}^2 + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{y\dot{y}}^2 + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial \dot{z}} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial \dot{z}}\right]\sigma_{y\dot{z}}^2 \\
& + \left[\frac{\partial \xi}{\partial z}\frac{\partial \zeta}{\partial \dot{x}} + \frac{\partial \zeta}{\partial z}\frac{\partial \xi}{\partial \dot{x}}\right]\sigma_{z\dot{x}}^2 + \left[\frac{\partial \xi}{\partial z}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial z}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{z\dot{y}}^2 + \left[\frac{\partial \xi}{\partial z}\frac{\partial \zeta}{\partial \dot{z}} + \frac{\partial \zeta}{\partial z}\frac{\partial \xi}{\partial \dot{z}}\right]\sigma_{z\dot{z}}^2 \\
& + \left[\frac{\partial \xi}{\partial \dot{x}}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial \dot{x}}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{\dot{x}\dot{y}}^2 + \left[\frac{\partial \xi}{\partial \dot{x}}\frac{\partial \zeta}{\partial \dot{z}} + \frac{\partial \zeta}{\partial \dot{x}}\frac{\partial \xi}{\partial \dot{z}}\right]\sigma_{\dot{x}\dot{z}}^2 + \left[\frac{\partial \xi}{\partial \dot{y}}\frac{\partial \zeta}{\partial \dot{z}} + \frac{\partial \zeta}{\partial \dot{y}}\frac{\partial \xi}{\partial \dot{z}}\right]\sigma_{\dot{y}\dot{z}}^2
\end{aligned}$$

For each spherical dimension, the different partial derivatives are given below, where we define

$$R_{2D} = (x^2 + y^2)^{1/2}$$

$$\mathcal{J}_R = \left[\frac{\partial R}{\partial x},\ \frac{\partial R}{\partial y},\ \frac{\partial R}{\partial z},\ \frac{\partial R}{\partial \dot{x}},\ \frac{\partial R}{\partial \dot{y}},\ \frac{\partial R}{\partial \dot{z}}\right]^T \tag{B.8}$$

$$= \left[xR^{-1},\ yR^{-1},\ zR^{-1},\ 0,\ 0,\ 0\right]^T \tag{B.9}$$

$$\mathcal{J}_\beta = \left[\frac{\partial \beta}{\partial x},\ \frac{\partial \beta}{\partial y},\ \frac{\partial \beta}{\partial z},\ \frac{\partial \beta}{\partial \dot{x}},\ \frac{\partial \beta}{\partial \dot{y}},\ \frac{\partial \beta}{\partial \dot{z}}\right]^T \tag{B.10}$$

$$= \left[yR_{2D}^{-2},\ -xR_{2D}^{-2},\ 0,\ 0,\ 0,\ 0\right]^T \tag{B.11}$$

$$\mathcal{J}_\epsilon = \left[\frac{\partial \epsilon}{\partial x},\ \frac{\partial \epsilon}{\partial y},\ \frac{\partial \epsilon}{\partial z},\ \frac{\partial \epsilon}{\partial \dot{x}},\ \frac{\partial \epsilon}{\partial \dot{y}},\ \frac{\partial \epsilon}{\partial \dot{z}}\right]^T \tag{B.12}$$

$$= \left[-xzR_{2D}^{-1}R^{-2},\ -yzR_{2D}^{-1}R^{-2},\ R_{2D}R^{-2},\ 0,\ 0,\ 0\right]^T \tag{B.13}$$

$$\mathcal{J}_{\dot{R}} = \left[\frac{\partial \dot{R}}{\partial x},\ \frac{\partial \dot{R}}{\partial y},\ \frac{\partial \dot{R}}{\partial z},\ \frac{\partial \dot{R}}{\partial \dot{x}},\ \frac{\partial \dot{R}}{\partial \dot{y}},\ \frac{\partial \dot{R}}{\partial \dot{z}}\right]^T \tag{B.14}$$

$$= \begin{bmatrix} \dot{x}R^2 - x(x\dot{x} + y\dot{y} + z\dot{z}) \\ \dot{y}R^2 - y(x\dot{x} + y\dot{y} + z\dot{z}) \\ \dot{z}R^2 - z(x\dot{x} + y\dot{y} + z\dot{z}) \\ xR^2 \\ yR^2 \\ zR^2 \end{bmatrix} R^{-3} \tag{B.15}$$

$$\mathcal{J}_{\dot{\beta}} = \left[\frac{\partial \dot{\beta}}{\partial x},\ \frac{\partial \dot{\beta}}{\partial y},\ \frac{\partial \dot{\beta}}{\partial z},\ \frac{\partial \dot{\beta}}{\partial \dot{x}},\ \frac{\partial \dot{\beta}}{\partial \dot{y}},\ \frac{\partial \dot{\beta}}{\partial \dot{z}}\right]^T \tag{B.16}$$

$$= \begin{bmatrix} -\dot{y}R_{2D}^2 - 2x(\dot{x}y - x\dot{y}) \\ \dot{x}R_{2D}^2 - 2y(\dot{x}y - x\dot{y}) \\ 0 \\ yR_{2D}^2 \\ -xR_{2D}^2 \\ 0 \end{bmatrix} R_{2D}^{-4} \tag{B.17}$$

$$\mathcal{J}_{\dot{\epsilon}} = \left[\frac{\partial \dot{\epsilon}}{\partial x}, \frac{\partial \dot{\epsilon}}{\partial y}, \frac{\partial \dot{\epsilon}}{\partial z}, \frac{\partial \dot{\epsilon}}{\partial \dot{x}}, \frac{\partial \dot{\epsilon}}{\partial \dot{y}}, \frac{\partial \dot{\epsilon}}{\partial \dot{z}}\right]^T \tag{B.18}$$

$$= \begin{bmatrix} xz(x\dot{x} + y\dot{y})R_{2D}^{-2} + (x\dot{z} - \dot{x}z) - 2x\dot{z}R_{2D}^2 R^{-2} + 2xz(x\dot{x} + y\dot{y})R^{-2} \\ yz(x\dot{x} + y\dot{y})R_{2D}^{-2} + (y\dot{z} - \dot{y}z) - 2y\dot{z}R_{2D}^2 R^{-2} + 2yz(x\dot{x} + y\dot{y})R^{-2} \\ -(x\dot{x} + y\dot{y}) - 2z\dot{z}R_{2D}^2 R^{-2} + 2z^2(x\dot{x} + y\dot{y})R^{-2} \\ -xz \\ -yz \\ R_{2D}^2 \end{bmatrix} R^{-2} R_{2D}^{-1} \tag{B.19}$$

B.2 Spherical to cartesian 3D conversion

In the way, based on the assumed available spherical information, the Cartesian coordinates and their derivatives are expressed as follows

$$x = R \sin\beta \cos\epsilon \tag{B.20}$$

$$y = R \cos\beta \cos\epsilon \tag{B.21}$$

$$z = R \sin\epsilon \tag{B.22}$$

$$\dot{x} = \dot{R} \sin\beta \cos\epsilon + R\dot{\beta} \cos\beta \cos\epsilon - R\dot{\epsilon} \sin\beta \sin\epsilon \tag{B.23}$$

$$\dot{y} = \dot{R} \cos\beta \cos\epsilon - R\dot{\beta} \sin\beta \cos\epsilon - R\dot{\epsilon} \cos\beta \sin\epsilon \tag{B.24}$$

$$\dot{z} = \dot{R} \sin\epsilon + R\dot{\epsilon} \cos\epsilon \tag{B.25}$$

and rewritten in a compact form as

$$\xi = \begin{bmatrix} \frac{\partial \xi}{\partial R} & \frac{\partial \xi}{\partial \beta} & \frac{\partial \xi}{\partial \epsilon} & \frac{\partial \xi}{\partial \dot{R}} & \frac{\partial \xi}{\partial \dot{\beta}} & \frac{\partial \xi}{\partial \dot{\epsilon}} \end{bmatrix} \times \begin{bmatrix} R \\ \beta \\ \epsilon \\ \dot{R} \\ \dot{\beta} \\ \dot{\epsilon} \end{bmatrix} \triangleq \mathcal{J}_\xi^T \boldsymbol{X} \tag{B.26}$$

which leads the following error variances/covariances expression

$$\sigma_{\xi\zeta}^2 = \mathcal{J}_\xi^T \begin{bmatrix} \sigma_R^2 & \sigma_{R\beta}^2 & \sigma_{R\epsilon}^2 & \sigma_{R\dot{R}}^2 & \sigma_{R\dot{\beta}}^2 & \sigma_{R\dot{\epsilon}}^2 \\ & \sigma_\beta^2 & \sigma_{\beta\epsilon}^2 & \sigma_{\beta\dot{R}}^2 & \sigma_{\beta\dot{\beta}}^2 & \sigma_{\beta\dot{\epsilon}}^2 \\ & & \sigma_\epsilon^2 & \sigma_{\epsilon\dot{R}}^2 & \sigma_{\epsilon\dot{\beta}}^2 & \sigma_{\epsilon\dot{\epsilon}}^2 \\ & & & \sigma_{\dot{R}}^2 & \sigma_{\dot{R}\dot{\beta}}^2 & \sigma_{\dot{R}\dot{\epsilon}}^2 \\ & & & & \sigma_{\dot{\beta}}^2 & \sigma_{\dot{\beta}\dot{\epsilon}}^2 \\ & & & & & \sigma_{\dot{\epsilon}}^2 \end{bmatrix} \mathcal{J}_\zeta$$

This can be rewritten using the partial derivatives as

$$\begin{aligned}\sigma_{\xi\zeta}^2 = {} & \frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial R}\sigma_R^2 + \frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\beta}\sigma_\beta^2 + \frac{\partial\xi}{\partial\epsilon}\frac{\partial\zeta}{\partial\epsilon}\sigma_\epsilon^2 + \left[\frac{\partial\xi}{\partial\dot{R}}\frac{\partial\zeta}{\partial\dot{R}}\right]\sigma_{\dot{R}}^2 + \frac{\partial\xi}{\partial\dot{\beta}}\frac{\partial\zeta}{\partial\dot{\beta}}\sigma_{\dot{\beta}}^2 + \frac{\partial\xi}{\partial\dot{\epsilon}}\frac{\partial\zeta}{\partial\dot{\epsilon}}\sigma_{\dot{\epsilon}}^2 \\
& + \left[\frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial\beta} + \frac{\partial\zeta}{\partial R}\frac{\partial\xi}{\partial\beta}\right]\sigma_{R\beta}^2 + \left[\frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial\epsilon} + \frac{\partial\zeta}{\partial R}\frac{\partial\xi}{\partial\epsilon}\right]\sigma_{R\epsilon}^2 + \left[\frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial\dot{R}} + \frac{\partial\zeta}{\partial R}\frac{\partial\xi}{\partial\dot{R}}\right]\sigma_{R\dot{R}}^2 \\
& + \left[\frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial\dot{\beta}} + \frac{\partial\zeta}{\partial R}\frac{\partial\xi}{\partial\dot{\beta}}\right]\sigma_{R\dot{\beta}}^2 + \left[\frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial\dot{\epsilon}} + \frac{\partial\zeta}{\partial R}\frac{\partial\xi}{\partial\dot{\epsilon}}\right]\sigma_{R\dot{\epsilon}}^2 + \left[\frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\epsilon} + \frac{\partial\zeta}{\partial\beta}\frac{\partial\xi}{\partial\epsilon}\right]\sigma_{\beta\epsilon}^2 \\
& + \left[\frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\dot{R}} + \frac{\partial\zeta}{\partial\beta}\frac{\partial\xi}{\partial\dot{R}}\right]\sigma_{\beta\dot{R}}^2 + \left[\frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\dot{\beta}} + \frac{\partial\zeta}{\partial\beta}\frac{\partial\xi}{\partial\dot{\beta}}\right]\sigma_{\beta\dot{\beta}}^2 + \left[\frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\dot{\epsilon}} + \frac{\partial\zeta}{\partial\beta}\frac{\partial\xi}{\partial\dot{\epsilon}}\right]\sigma_{\beta\dot{\epsilon}}^2 \\
& + \left[\frac{\partial\xi}{\partial\epsilon}\frac{\partial\zeta}{\partial\dot{R}} + \frac{\partial\zeta}{\partial\epsilon}\frac{\partial\xi}{\partial\dot{R}}\right]\sigma_{\epsilon\dot{R}}^2 + \left[\frac{\partial\xi}{\partial\epsilon}\frac{\partial\zeta}{\partial\dot{\beta}} + \frac{\partial\zeta}{\partial\epsilon}\frac{\partial\xi}{\partial\dot{\beta}}\right]\sigma_{\epsilon\dot{\beta}}^2 + \left[\frac{\partial\xi}{\partial\epsilon}\frac{\partial\zeta}{\partial\dot{\epsilon}} + \frac{\partial\zeta}{\partial\epsilon}\frac{\partial\xi}{\partial\dot{\epsilon}}\right]\sigma_{\epsilon\dot{\epsilon}}^2 \\
& + \left[\frac{\partial\xi}{\partial\dot{R}}\frac{\partial\zeta}{\partial\dot{\beta}} + \frac{\partial\zeta}{\partial\dot{R}}\frac{\partial\xi}{\partial\dot{\beta}}\right]\sigma_{\dot{R}\dot{\beta}}^2 + \left[\frac{\partial\xi}{\partial\dot{R}}\frac{\partial\zeta}{\partial\dot{\epsilon}} + \frac{\partial\zeta}{\partial\dot{R}}\frac{\partial\xi}{\partial\dot{\epsilon}}\right]\sigma_{\dot{R}\dot{\epsilon}}^2 + \left[\frac{\partial\xi}{\partial\dot{\beta}}\frac{\partial\zeta}{\partial\dot{\epsilon}} + \frac{\partial\zeta}{\partial\dot{\beta}}\frac{\partial\xi}{\partial\dot{\epsilon}}\right]\sigma_{\dot{\beta}\dot{\epsilon}}^2\end{aligned}$$

where the different partial derivatives are given below.

$$\mathcal{J}_x = \left[\frac{\partial x}{\partial R},\ \frac{\partial x}{\partial\beta},\ \frac{\partial x}{\partial\epsilon},\ \frac{\partial x}{\partial\dot{R}},\ \frac{\partial x}{\partial\dot{\beta}},\ \frac{\partial x}{\partial\dot{\epsilon}}\right]^T = \begin{bmatrix}\sin\beta\cos\epsilon \\ R\cos\beta\cos\epsilon \\ -R\sin\beta\sin\epsilon \\ 0 \\ 0 \\ 0\end{bmatrix} \tag{B.27}$$

$$\mathcal{J}_y = \left[\frac{\partial y}{\partial R},\ \frac{\partial y}{\partial\beta},\ \frac{\partial y}{\partial\epsilon},\ \frac{\partial y}{\partial\dot{R}},\ \frac{\partial y}{\partial\dot{\beta}},\ \frac{\partial y}{\partial\dot{\epsilon}}\right]^T = \begin{bmatrix}\cos\beta\cos\epsilon \\ -R\sin\beta\cos\epsilon \\ -R\cos\beta\sin\epsilon \\ 0 \\ 0 \\ 0\end{bmatrix} \tag{B.28}$$

$$\mathcal{J}_z = \left[\frac{\partial z}{\partial R},\ \frac{\partial z}{\partial\beta},\ \frac{\partial z}{\partial\epsilon},\ \frac{\partial z}{\partial\dot{R}},\ \frac{\partial z}{\partial\dot{\beta}},\ \frac{\partial z}{\partial\dot{\epsilon}}\right]^T = \begin{bmatrix}\sin\epsilon \\ 0 \\ R\cos\epsilon \\ 0 \\ 0 \\ 0\end{bmatrix} \tag{B.29}$$

$$\mathcal{J}_{\dot{x}} = \left[\frac{\partial \dot{x}}{\partial R}, \frac{\partial \dot{x}}{\partial \beta}, \frac{\partial \dot{x}}{\partial \epsilon}, \frac{\partial \dot{x}}{\partial \dot{R}}, \frac{\partial \dot{x}}{\partial \dot{\beta}}, \frac{\partial \dot{x}}{\partial \dot{\epsilon}}\right]^T \tag{B.30}$$

$$= \left[\frac{\partial \dot{x}}{\partial R}, \frac{\partial \dot{x}}{\partial \beta}, \frac{\partial \dot{x}}{\partial \epsilon}, \frac{\partial x}{\partial R}, \frac{\partial x}{\partial \beta}, \frac{\partial x}{\partial \epsilon}\right]^T \tag{B.31}$$

$$= \begin{bmatrix} \dot{\beta}\cos\beta\cos\epsilon - \dot{\epsilon}\sin\beta\sin\epsilon \\ \dot{R}\cos\beta\cos\epsilon - \dot{\beta}R\sin\beta\cos\epsilon - \dot{\epsilon}R\cos\beta\sin\epsilon \\ -\dot{R}\sin\beta\sin\epsilon - \dot{\beta}R\cos\beta\sin\epsilon - \dot{\epsilon}R\sin\beta\cos\epsilon \\ \sin\beta\cos\epsilon \\ R\cos\beta\cos\epsilon \\ -R\sin\beta\sin\epsilon \end{bmatrix} \tag{B.32}$$

$$\mathcal{J}_{\dot{y}} = \left[\frac{\partial \dot{y}}{\partial R}, \frac{\partial \dot{y}}{\partial \beta}, \frac{\partial \dot{y}}{\partial \epsilon}, \frac{\partial \dot{y}}{\partial \dot{R}}, \frac{\partial \dot{y}}{\partial \dot{\beta}}, \frac{\partial \dot{y}}{\partial \dot{\epsilon}}\right]^T \tag{B.33}$$

$$= \left[\frac{\partial \dot{y}}{\partial R}, \frac{\partial \dot{y}}{\partial \beta}, \frac{\partial \dot{y}}{\partial \epsilon}, \frac{\partial y}{\partial R}, \frac{\partial y}{\partial \beta}, \frac{\partial y}{\partial \epsilon}\right]^T \tag{B.34}$$

$$= \begin{bmatrix} -\dot{\beta}\sin\beta\cos\epsilon - \dot{\epsilon}\cos\beta\sin\epsilon \\ -\dot{R}\sin\beta\cos\epsilon - \dot{\beta}R\cos\beta\cos\epsilon + \dot{\epsilon}R\sin\beta\sin\epsilon \\ -\dot{R}\cos\beta\sin\epsilon + \dot{\beta}R\sin\beta\sin\epsilon - \dot{\epsilon}R\cos\beta\cos\epsilon \\ \cos\beta\cos\epsilon \\ -R\sin\beta\cos\epsilon \\ -R\cos\beta\sin\epsilon \end{bmatrix} \tag{B.35}$$

$$\mathcal{J}_{\dot{z}} = \left[\frac{\partial \dot{z}}{\partial R}, \frac{\partial \dot{z}}{\partial \beta}, \frac{\partial \dot{z}}{\partial \epsilon}, \frac{\partial \dot{z}}{\partial \dot{R}}, \frac{\partial \dot{z}}{\partial \dot{\beta}}, \frac{\partial \dot{z}}{\partial \dot{\epsilon}}\right]^T \tag{B.36}$$

$$= \left[\frac{\partial \dot{z}}{\partial R}, \frac{\partial \dot{z}}{\partial \beta}, \frac{\partial \dot{z}}{\partial \epsilon}, \frac{\partial z}{\partial R}, \frac{\partial z}{\partial \beta}, \frac{\partial z}{\partial \epsilon}\right]^T \tag{B.37}$$

$$= \begin{bmatrix} \dot{\epsilon}\cos\epsilon \\ 0 \\ \dot{R}\cos\epsilon - \dot{\epsilon}R\sin\epsilon \\ \sin\epsilon \\ 0 \\ R\cos\epsilon \end{bmatrix} \tag{B.38}$$

B.3 Cartesian 2D to polar

The polar coordinate are a 2D special case of the spherical coordinates. The coodinate transformations from Cartesian (x, y) to polar (R, β) are derived from the spherical as

follows

$$R = \sqrt{x^2 + y^2} \tag{B.39}$$

$$\beta = \tan^{-1} \frac{x}{y} \tag{B.40}$$

$$\dot{R} = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} \tag{B.41}$$

$$\dot{\beta} = \frac{y\dot{x} - x\dot{y}}{x^2 + y^2} \tag{B.42}$$

The above given variables can be rewritten as

$$\xi = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} & \frac{\partial \xi}{\partial \dot{x}} & \frac{\partial \xi}{\partial \dot{y}} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \triangleq \mathcal{J}_\xi^T \boldsymbol{X} \tag{B.43}$$

and, as in the previous case,

$$\sigma_{\xi\zeta}^2 = \mathcal{J}_\xi^T \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 & \sigma_{x\dot{x}}^2 & \sigma_{x\dot{y}}^2 \\ & \sigma_y^2 & \sigma_{y\dot{x}}^2 & \sigma_{y\dot{y}}^2 \\ & & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}}^2 \\ & & & \sigma_{\dot{y}}^2 \end{bmatrix} \mathcal{J}_\zeta$$

which can be rewritten using the partial derivatives as

$$\begin{aligned} \sigma_{\xi\zeta}^2 = & \frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial x}\sigma_x^2 + \frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial y}\sigma_y^2 + \frac{\partial \xi}{\partial \dot{x}}\frac{\partial \zeta}{\partial \dot{x}}\sigma_{\dot{x}}^2 + \frac{\partial \xi}{\partial \dot{y}}\frac{\partial \zeta}{\partial \dot{y}}\sigma_{\dot{y}}^2 \\ & + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial y} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial y}\right]\sigma_{xy}^2 + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial \dot{x}} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial \dot{x}}\right]\sigma_{x\dot{x}}^2 + \left[\frac{\partial \xi}{\partial \dot{x}}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial \dot{x}}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{\dot{x}\dot{y}}^2 \\ & + \left[\frac{\partial \xi}{\partial x}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial x}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{x\dot{y}}^2 + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial \dot{x}} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial \dot{x}}\right]\sigma_{y\dot{x}}^2 + \left[\frac{\partial \xi}{\partial y}\frac{\partial \zeta}{\partial \dot{y}} + \frac{\partial \zeta}{\partial y}\frac{\partial \xi}{\partial \dot{y}}\right]\sigma_{y\dot{y}}^2 \end{aligned}$$

where the partial derivatives, of each dimension, are given below.

$$\mathcal{J}_R = \left[\frac{\partial R}{\partial x},\ \frac{\partial R}{\partial y},\ \frac{\partial R}{\partial \dot{x}},\ \frac{\partial R}{\partial \dot{y}}\right]^T \tag{B.44}$$

$$= \left[xR^{-1},\ yR^{-1},\ 0,\ 0\right]^T \tag{B.45}$$

$$\mathcal{J}_\beta = \left[\frac{\partial \beta}{\partial x},\ \frac{\partial \beta}{\partial y},\ \frac{\partial \beta}{\partial \dot{x}},\ \frac{\partial \beta}{\partial \dot{y}}\right]^T \tag{B.46}$$

$$= \left[yR^{-2},\ -xR^{-2},\ 0,\ 0\right]^T \tag{B.47}$$

$$\mathcal{J}_{\dot{R}} = \left[\frac{\partial \dot{R}}{\partial x}, \frac{\partial \dot{R}}{\partial y}, \frac{\partial \dot{R}}{\partial \dot{x}}, \frac{\partial \dot{R}}{\partial \dot{y}}\right]^T \tag{B.48}$$

$$= \begin{bmatrix} \dot{x}R^2 - x^2\dot{x} - xy\dot{y} \\ \dot{y}R^2 - xy\dot{x} - y^2\dot{y} \\ xR^2 \\ yR^2 \end{bmatrix} R^{-3} \tag{B.49}$$

$$\mathcal{J}_{\dot{\beta}} = \left[\frac{\partial \dot{\beta}}{\partial x}, \frac{\partial \dot{\beta}}{\partial y}, \frac{\partial \dot{\beta}}{\partial \dot{x}}, \frac{\partial \dot{\beta}}{\partial \dot{y}}\right]^T \tag{B.50}$$

$$= \begin{bmatrix} -\dot{y}R^2 - 2xy\dot{x} + 2x^2\dot{y} \\ \dot{x}R^2 - 2y^2\dot{x} + 2xy\dot{y} \\ yR^2 \\ -xR^2 \end{bmatrix} R^{-4} \tag{B.51}$$

B.4 Polar to cartesian 2D

In the 2D case, the Cartesian coordinates and their derivatives are obtained as follows:

$$x = R\sin\beta \tag{B.52}$$

$$y = R\cos\beta \tag{B.53}$$

$$\dot{x} = \dot{R}\sin\beta + \dot{\beta}R\cos\beta \tag{B.54}$$

$$\dot{y} = \dot{R}\cos\beta - \dot{\beta}R\sin\beta \tag{B.55}$$

Each of the above given variables can be rewritten as

$$\xi = \begin{bmatrix} \frac{\partial \xi}{\partial R} & \frac{\partial \xi}{\partial \beta} & \frac{\partial \xi}{\partial \dot{R}} & \frac{\partial \xi}{\partial \dot{\beta}} \end{bmatrix} \times \begin{bmatrix} R \\ \beta \\ \dot{R} \\ \dot{\beta} \end{bmatrix} \triangleq \mathcal{J}_\xi^T \boldsymbol{X} \tag{B.56}$$

with as a variance/covariance matrix

$$\sigma^2_{\xi\zeta} = \mathcal{J}_\xi^T \begin{bmatrix} \sigma^2_R & \sigma^2_{R\beta} & \sigma^2_{R\dot{R}} & \sigma^2_{R\dot{\beta}} \\ & \sigma^2_\beta & \sigma^2_{\beta\dot{R}} & \sigma^2_{\beta\dot{\beta}} \\ & & \sigma^2_{\dot{R}} & \sigma^2_{\dot{R}\dot{\beta}} \\ & & & \sigma^2_{\dot{\beta}} \end{bmatrix} \mathcal{J}_\zeta$$

whose development is given by

$$\begin{aligned} \sigma^2_{\xi\zeta} = {} & \frac{\partial \xi}{\partial R}\frac{\partial \zeta}{\partial R}\sigma^2_R + \frac{\partial \xi}{\partial \beta}\frac{\partial \zeta}{\partial \beta}\sigma^2_\beta + \frac{\partial \xi}{\partial \dot{R}}\frac{\partial \zeta}{\partial \dot{R}}\sigma^2_{\dot{R}} + \frac{\partial \xi}{\partial \dot{\beta}}\frac{\partial \zeta}{\partial \dot{\beta}}\sigma^2_{\dot{\beta}} \\ & + \left[\frac{\partial \xi}{\partial R}\frac{\partial \zeta}{\partial \beta} + \frac{\partial \zeta}{\partial R}\frac{\partial \xi}{\partial \beta}\right]\sigma^2_{R\beta} + \left[\frac{\partial \xi}{\partial R}\frac{\partial \zeta}{\partial \dot{R}} + \frac{\partial \zeta}{\partial R}\frac{\partial \xi}{\partial \dot{R}}\right]\sigma^2_{R\dot{R}} + \left[\frac{\partial \xi}{\partial R}\frac{\partial \zeta}{\partial \dot{\beta}} + \frac{\partial \zeta}{\partial R}\frac{\partial \xi}{\partial \dot{\beta}}\right]\sigma^2_{R\dot{\beta}} \\ & + \left[\frac{\partial \xi}{\partial \beta}\frac{\partial \zeta}{\partial \dot{R}} + \frac{\partial \zeta}{\partial \beta}\frac{\partial \xi}{\partial \dot{R}}\right]\sigma^2_{\beta\dot{R}} + \left[\frac{\partial \xi}{\partial \beta}\frac{\partial \zeta}{\partial \dot{\beta}} + \frac{\partial \zeta}{\partial \beta}\frac{\partial \xi}{\partial \dot{\beta}}\right]\sigma^2_{\beta\dot{\beta}} + \left[\frac{\partial \xi}{\partial \dot{R}}\frac{\partial \zeta}{\partial \dot{\beta}} + \frac{\partial \zeta}{\partial \dot{R}}\frac{\partial \xi}{\partial \dot{\beta}}\right]\sigma^2_{\dot{R}\dot{\beta}} \end{aligned}$$

where the Jacobians are given by

$$\mathcal{J}_x = \begin{bmatrix}\sin\beta, & R\cos\beta, & 0, & 0\end{bmatrix}^T \tag{B.57}$$

$$\mathcal{J}_y = \begin{bmatrix}\cos\beta, & -R\sin\beta, & 0, & 0\end{bmatrix}^T \tag{B.58}$$

$$\mathcal{J}_{\dot{x}} = \begin{bmatrix}\dot{\beta}\cos\beta, & \dot{R}\cos\beta - \dot{\beta}R\sin\beta, & \sin\beta, & R\cos\beta\end{bmatrix}^T \tag{B.59}$$

$$\mathcal{J}_{\dot{y}} = \begin{bmatrix}-\dot{\beta}\sin\beta, & -\dot{R}\sin\beta - \dot{\beta}R\cos\beta, & \cos\beta, & -R\sin\beta\end{bmatrix}^T \tag{B.60}$$

B.5 Polar contact to cartesian

In the case of polar contacts, the only available information is the range R and the beating β, *i.e.*, no information about the rates is available. Hence the possible transformations are

$$x = R\sin\beta \tag{B.61}$$

$$y = R\cos\beta \tag{B.62}$$

This reduces the above-given notation to

$$\xi = \begin{bmatrix}\frac{\partial\xi}{\partial R} & \frac{\partial\xi}{\partial\beta}\end{bmatrix} \times \begin{bmatrix}R \\ \beta\end{bmatrix} \triangleq \mathcal{J}_\xi^T \boldsymbol{X} \tag{B.63}$$

with as a variance/covariance matrix

$$\sigma_{\xi\zeta}^2 = \mathcal{J}_\xi^T \begin{bmatrix}\sigma_R^2 & \sigma_{R\beta}^2 \\ \sigma_{\beta R}^2 & \sigma_\beta^2\end{bmatrix} \mathcal{J}_\zeta$$

Since

$$\sigma_{R\beta}^2 = \sigma_{\beta R}^2 = 0$$

this reduces to

$$\sigma_{\xi\zeta}^2 = \frac{\partial\xi}{\partial R}\frac{\partial\zeta}{\partial R}\sigma_R^2 + \frac{\partial\xi}{\partial\beta}\frac{\partial\zeta}{\partial\beta}\sigma_\beta^2$$

which gives for x and y

$$\sigma_x^2 = \sin^2\beta\sigma_R^2 + R^2\cos^2\beta\sigma_\beta^2 \tag{B.64}$$

$$\sigma_y^2 = \cos^2\beta\sigma_R^2 + R^2\sin^2\beta\sigma_\beta^2 \tag{B.65}$$

$$\sigma_{xy}^2 = \sin\beta\cos\beta(\sigma_R^2 - R^2\sigma_\beta^2) \tag{B.66}$$

This page intentionally left blank.

Distribution list

This page intentionally left blank.

Distribution list

DRDC Valcartier TR 2003-284

Internal distribution

1 Director General

3 Document Library

1 Head/Decision Support Systems (author)

1 Dr R. Breton

1 Dr P. Valin

1 Dr A. Guitouni

1 Head/Information and Knowledge Management

1 R/Visualization & Geo-spatial Systems

1 R/Knowledge Management Systems

1 R/Informations System Warfare

1 Head/System of Systems

1 R/Metrics & Experimentation

1 R/Distributed Synthetic Environment

1 R/System Engineering & Architecture

1 LtCol. B. Carrier

1 LCdr É. Tremblay

1 Maj. B. Deschênes

1 Dr M. Allouche

1 M. Bélanger

1 Dr A. Benaskeur (author)

1 J. Berger

1 M. Blanchette

1 Dr A. Boukhtouta

1 Dr A.-C. Bourry-Brisset

1 C. Daigle

1 Dr H. Irandoust

1 Dr A.-L. Jousselme

1 P. Maupin

1 S. Paradis

1 F. Rhéaume (author)

1 A. Sahi

1 J.M.J. Roy

1 A. Jouan

Total internal copies: 35

External distribution

1 M. C. Florea
Informatique WGZ inc.

1 DRD KIM (PDF file)

2 Director Science & Technology Maritime(DSTM)

2 Director Science & Technology Land (DSTL)

2 Director Science & Technology Air (DSTA)

2 Director Science & Technology C4ISR (DSTC4ISR)

1 Director Maritime Requirements Sea (DMRS) 4

1 Director Maritime Requirements Sea (DMRS) 6

1 Director Aerospace Requirements (DAR) 4

1 Director Aerospace Requirements (DAR) 4-2

2 Director Maritime Ship Support (DMSS) 6

2 Director Maritime Ship Support (DMSS) 8

2 DRDC - Atlantic:
Attn: Dr. Bruce MacArthur
Attn: Dr. Jim S. Kennedy

2 DRDC - Ottawa:
Attn: Barbara Ford
Attn: Dan Brookes

2 CF Maritime Warfare School CFB Halifax
Halifax, Nova Scotia
Attn: TAC AAW
OIC Modeling and Simulation

2 Canadian Forces Naval Operations School CFB Halifax
Halifax, Nova Scotia
Attn: Tactic
CT AWW

1 Canadian Forces Naval Engineering School CFB Halifax
Halifax, Nova Scotia
Attn: CSTC

1 Operational Requirements Analysis Cell CFB Halifax
Halifax, Nova Scotia
Attn: Commanding Officer

1 Canadian Forces Fleet School CFB Esquimalt
Esquimalt, British Columbia
Attn: Commanding Officer/WTD

1 Operational Requirements Analysis Cell CFB Esquimalt
Esquimalt, British Columbia
Attn: Commanding Officer

Total external copies: 30

Total copies: 65

FICHE DE CONTRÔLE DU DOCUMENT

1. PROVENANCE (le nom et l'adresse)
François Rhéaume
RDDC - Valcartier
2495, boul. Pie-Xi Nord
Québec Qc G3J 1X5 Canada

2. COTE DE SÉCURITÉ
(y compris les notices d'avertissement, s'il y a lieu)
UNCLASSIFIED

3. TITRE (Indiquer la cote de sécurité au moyen de l'abréviation (S, C, R ou U) mise entre parenthèses, immédiatement après le titre.)

Impact of the tactical picture quality on the fire control radar search-lock-on time (U)

4. AUTEURS (Nom de famille, prénom et initiales. Indiquer les grades militaires, ex.: Bleau, Maj. Louis E.)
François Rhéaume et Abder Rezak Benaskeur

5. DATE DE PUBLICATION DU DOCUMENT (mois et année)
2006

6a. NOMBRE DE PAGES
42

6b. NOMBRE DE REFERENCES
2

7. DESCRIPTION DU DOCUMENT (La catégorie du document, par exemple rapport, note technique ou mémorandum. Indiquer les dates lorsque le rapport couvre une période définie.)
Rapport technique

8. PARRAIN (le nom et l'adresse)

9a. NUMÉRO DU PROJET OU DE LA SUBVENTION
(Spécifier si c'est un projet ou une subvention)

9b. NUMÉRO DE CONTRAT

10a. NUMÉRO DU DOCUMENT DE L'ORGANISME EXPÉDITEUR
TR 2003-284

10b. AUTRES NUMÉROS DU DOCUMENT
N/A

11. ACCÈS AU DOCUMENT (Toutes les restrictions concernant une diffusion plus ample du document, autres que celles inhérentes à la cote de sécurité.)

- [x] Diffusion illimitée
- [ ] Diffusion limitée aux entrepreneurs des pays suivants (spécifier)
- [ ] Diffusion limitée aux entrepreneurs canadiens (avec une justification)
- [ ] Diffusion limitée aux organismes gouvernementaux (avec une justification)
- [ ] Diffusion limitée aux ministères de la Défense
- [ ] Autres

12. ANNONCE DU DOCUMENT (Toutes les restrictions à l'annonce bibliographique de ce document. Cela correspond, en principe, aux données d'accès au document (11). Lorsqu'une diffusion supplémentaire (à d'autres organismes que ceux précisés à la case 11) est possible, on pourra élargir le cercle de diffusion de l'annonce.)

13. SOMMAIRE (Un résumé clair et concis du document. Les renseignements peuvent aussi figurer ailleurs dans le document. Il est souhaitable que le sommaire des documents classifiés soit non classifié. Il faut inscrire au commencement de chaque paragraphe du sommaire la cote de sécurité applicable aux renseignements qui s'y trouvent, à moins que le document lui-même soit non classifié. Se servir des lettres suivantes: (S), (C), (R) ou (U). Il n'est pas nécessaire de fournir ici des sommaires dans les deux langues officielles à moins que le document soit bilingue.)

Data fusion is suitable for a broad range of decision support applications. To cope with a larger class of problems and contexts, data fusion gains to be adaptive. Adaptation in data fusion corresponds to Level 4 of the JDL model, also referred to as process refinement. The Decision Support Systems Section (DSS) at Defence Research \& Development Canada (DRDC)~--~Valcartier has initiated research activities aiming at developing and demonstrating advanced concepts of adaptive data fusion that could apply to the current Halifax and Iroquois Class Command \& Control Systems (CCS), as well as their possible future upgrades, in order to improve their performance against the predicted future threats. This document gives a brief description of the adaptive data fusion concepts. It also presents a new Measure Of Effectiveness (MOE) that serves as an adaptation trigger in the target-tracking problem in maritime Above Water Warfare (AWW) applications. The proposed MOE uses the search to lock-on time of the Fire Control Radar (FCR) and aims at establishing and quantifying the effect of the quality of the Maritime Tactical Picture (MTP) on the diminution of battle space size and reaction time. Besides adaptation of the sensing and processing operation, this MOE allows addressing the trade-off between the time dedicated to the tracking with surveillance radars versus the time spent in search and lock-on with FCR.

14. MOTS-CLÉS, DESCRIPTEURS OU RENSEIGNEMENTS SPÉCIAUX (Expressions ou mots significatifs du point de vue technique, qui caractérisent un document et peuvent aider à le cataloguer. Il faut choisir des termes qui n'exigent pas de cote de sécurité. Des renseignements tels que le modèle de l'équipement, la marque de fabrique, le nom de code du projet militaire, la situation géographique, peuvent servir de mots-clés. Si possible, on doit choisir des mots-clés d'un thésaurus, par exemple le "Thesaurus of Engineering and Scientific Terms (TESTS)". Nommer ce thésaurus. Si l'on ne peut pas trouver de termes non classifiés, il faut indiquer la classification de chaque terme comme on le fait avec le titre.)

Data fusion, Tracking, Fire Control Radar, Hand-over, Adaptation

Defence R&D Canada

Canada's Leader in Defence
and National Security
Science and Technology

R & D pour la défense Canada

Chef de file au Canada en matière
de science et de technologie pour
la défense et la sécurité nationale

DEFENCE DÉFENSE

WWW.drdc-rddc.gc.ca